COMPUTATION OF NATURAL CONVECTION IN A POROUS PARALLELOGRAMMIC ENCLOSURE WITH A MAGNETIC FIELD

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Abstract

Detailed numerical calculations are presented in this paper for natural convection in a porous parallelogrammic enclosure with a magnetic field. The inclined walls are maintained isothermally at different temperatures. The top and bottom horizontal straight walls are kept adiabatic. To simplify the effort in matching the grid mesh with the inclined walls of the parallelogrammic region/geometry, the computational domain is mapped onto a rectangular shape using a non-linear axis transformation. Transport equations are modeled by a stream-vorticity formulation then expressed in the new coordinate system and solved numerically by a finite difference method. Based upon the numerical predictions, we found the convection modes within the enclosure depended upon the Rayleigh number and the inclination angle. As the value of magnetic field is made larger, the strength of the heat transfer is progressively suppressed. Tuning the inclination angle decreases the heat transfer performance.

Keywords: Darcy’s law, natural convection, numerical method, Porous media

1. Introduction

Convective flows in porous media have occupied the central stage in many fundamental heat transfer analyses and received considerable attention over the last few decades. This interest is due to their wide range of applications, for example, high performance insulation for buildings, chemical catalytic reactors, packed sphere beds, grain storage and geophysical problems, such as frost heave. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors, and geothermal energy systems [1].

Most of the published papers are concerned with the analysis of natural convection in square/rectangular enclosures filled with porous media; see, for example, Saeid and Pop [2], Oztop [3], Kumari and Nath [4] and Kaluri and Basak [5]. In reality, the shape of the enclosure can also be non-rectangular in practical engineering applications, such as solar collectors or heat exchangers with different duct constructions. The study of convective flow in a non-rectangular geometry is more difficult than that of square or rectangular enclosures due to the presence of sloping walls. In general, the mesh nodes do not lie along the sloping walls and consequently, from a programming and computational point of view, the effort required for determining flow characteristic increases significantly. Relatively little work has been done in a porous parallelogrammic geometry. Baytas and Pop [6] applied an alternative direction implicit (ADI) finite difference method to solve natural convection in a porous parallelogrammic enclosure. However, they did not consider effect of a magnetic field.

Bian et al. [7] firstly investigated the effect of a transverse magnetic field on natural convection in an inclined porous rectangular enclosure. Then Wang et al. [8] extended Darcy model in [7] to the Brinkman-Forchheimer model. Bian study was later extended by Grosan et al. [9] to include a magnetic field, inclined at an angle with respect to the horizontal plane. The angles’ ranges are between 0 to π/2, 0 corresponds to a horizontal magnetic field and π/2 corresponds to a vertical magnetic field. Recently Mansour et al. [10] studied effect of magnetic field when internal heat generation exists inside the porous material. To the best of our knowledge, investigation of the effects of a magnetic field on natural convection in a parallelogrammic enclosure has not been undertaken yet. The purpose of the present paper is therefore to investigate the effects of a magnetic field on steady natural convection in a porous parallelogrammic enclosure. To do
it the computational domain is mapped onto a rectangular shaped cavity using a nonlinear axis transformation as explained by Baytas and Pop [6].

2. Methods

We consider the steady, two-dimensional natural convection flow in a parallelogrammic region filled with an electrically conducting fluid-saturated porous medium, see Fig. 1(a). The co-ordinate system employed is also depicted in this figure. The top and bottom surfaces of the convective region are assumed to be thermally insulated and the sloping surfaces to be heated and cooled at constant temperatures \( T_h \) and \( T_c \), respectively. \( \theta_s \) is the inclination angle of the sloping walls. \( \theta_s = 0^\circ \) means that enclosure is a square with width \( L \).

A uniform and constant magnetic field with the magnitude \( B_0 \) is applied in the horizontal direction. The viscous, radiation and Joule heating effects are neglected. The resulting convective flow is governed by the combined mechanism of the driven buoyancy force and the retarding effect of the magnetic field. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in favor of the applied magnetic field.

Under the above assumptions, the conservation equations for momentum and energy in stream-vorticity formulation for steady two-dimensional natural convection flow can be written as:

\[
\begin{align*}
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= -gK \beta \frac{\partial T}{\partial x} - \frac{\sigma KB_0^2}{\mu} \frac{\partial^2 \psi}{\partial x^2} \\
\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} &= \alpha_m \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\end{align*}
\]

where \( x \) and \( y \) are the Cartesian coordinates measured in the horizontal and vertical directions respectively, \( g \) is the acceleration due to gravity, \( K \) is the permeability of the porous medium, \( \beta \) is the coefficient of thermal expansion, \( \nu \) is the kinematic viscosity, \( \sigma \) is the electrical conductivity, \( \mu \) is the dynamic viscosity, \( \alpha_m \) is the thermal diffusivity. \( T \) is the fluid temperature variable. \( \psi \) is the stream function defined from the velocity components \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \). Eq. (1) corresponds to the porous medium modeled according to Darcy’s law. Because of the complexity of pore geometries in a porous medium, Darcy’s law has to be used to obtain any meaningful insights into the physics of flow in porous media.

In general, no rectangular grid mesh can be generated that fits all four surfaces. However, the computational domain can be mapped onto a rectangular domain as shown in Fig. 1(b) using the following transformation:

\[
\bar{x} = x - y \tan \theta_s, \quad \bar{y} = y
\]

Note that using this transformation one has

\[
\frac{\partial}{\partial x} = \frac{\partial}{\partial \bar{x}}, \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial \bar{y}} - \tan \theta_s \frac{\partial}{\partial \bar{x}}
\]

This implies:

\[
\begin{align*}
\frac{\partial}{\partial \bar{x}^2} &= \frac{\partial}{\partial x^2} \\
\frac{\partial^2}{\partial \bar{y}^2} &= \frac{\partial^2}{\partial y^2} - \tan \theta_s \left( 2 \frac{\partial^2}{\partial y \partial x} - \tan \theta_s \frac{\partial^2}{\partial x^2} \right)
\end{align*}
\]
With Eqs. (3)–(5), the Eqs. (1)–(2) become:

\[
\frac{\partial^2 \psi}{\partial x^2} - 2 \sin \theta \cos \theta \frac{\partial \psi}{\partial y} + \cos^2 \theta \frac{\partial^2 \psi}{\partial y^2} = -\cos^2 \theta \left( \frac{gK \beta}{\nu} \frac{\partial T}{\partial x} + \frac{\sigma KB_0}{\mu} \frac{\partial^2 \psi}{\partial y^2} \right) \tag{6}
\]

\[
\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{\alpha_v}{\nu} \cos^2 \theta \left( \frac{\partial^2 T}{\partial x^2} - 2 \sin \theta \cos \theta \frac{\partial^2 \psi}{\partial y^2} + \cos^2 \theta \frac{\partial^2 T}{\partial y^2} \right) \tag{7}
\]

Further, we introduce the following dimensionless variable:

\[
\xi = \frac{x}{a}, \quad \eta = \frac{y}{L \cos \theta}, \quad \Psi = \frac{\psi}{\alpha_v}, \quad \Theta = \frac{T - T_o}{(T_h - T_o)} \tag{8}
\]

Expressed in these variables, Eqs. (6)–(7) transform to

\[
\left( 1 + \cos^2 \theta Ha \right) \frac{\partial^2 \Psi}{\partial \xi^2} - 2 \sin \theta \cos \theta \frac{\partial \Psi}{\partial \eta} + \frac{1}{A^2} \frac{\partial^2 \Psi}{\partial \eta^2} = -Ra \cos^2 \theta \frac{\partial \Theta}{\partial \xi} \tag{9}
\]

\[
\frac{\partial \Psi}{\partial \xi} \frac{\partial \Theta}{\partial \eta} - \frac{\partial \Psi}{\partial \eta} \frac{\partial \Theta}{\partial \xi} = A \cos \theta \eta \frac{\partial \theta}{\partial \xi} \frac{\partial \eta}{\partial \xi} \frac{\partial \eta}{\partial \eta} \left( \frac{\partial^2 \Theta}{\partial \xi^2} - 2 \sin \theta \cos \theta \frac{\partial^2 \Theta}{\partial \eta^2} + \frac{1}{A^2} \frac{\partial \Theta}{\partial \eta^2} \right) \tag{10}
\]

Where \( A = L/a \) is the enclosure aspect ratio, \( Ra = gK \beta L \Delta T / (\alpha_v \nu) \) is the Rayleigh number and \( Ha = \sigma KB_0^2 / \mu \) is the Hartmann number for the porous medium.

Using (3) and (8), the relevant hydrodynamic and thermal boundary of Eqs. (9)–(10) can be written as:

\[
\Psi = 0, \quad \Theta = 1 \text{ on } \xi = 0 \tag{11}
\]

\[
\Psi = 0, \quad \Theta = 0 \text{ on } \xi = 1 \tag{11}
\]

\[
\Psi = 0, \quad \frac{\partial \Theta}{\partial \eta} - A \sin \theta \frac{\partial \Theta}{\partial \xi} = 0 \text{ on } \eta = 0, 1 \tag{11}
\]

To measure the heat transfer performance we define average Nusselt number as

\[
Nu = \frac{1}{\cos \theta} \int_0^1 \frac{1}{A} \left( \frac{\sin \theta}{\eta} \frac{\partial \Theta}{\partial \eta} - \frac{\partial \Theta}{\partial \xi} \right) \eta^2 \tag{12}
\]

We employed finite difference method to solve Eqs. (9)–(10) subject to the boundary conditions Eq. (11). The central difference method is applied for discretizing the equations. Next, the solution of the algebraic equations was performed using Gauss-Seidel iteration with relaxation. The unknowns \( \Theta \) and \( \psi \) are calculated until the following convergence criterion is fulfilled:

\[
\max \left[ \frac{1}{\xi_{i,j}} \left| \frac{\zeta_{n+1} - \zeta_{n}}{\xi_{i,j}} \right| \right] \leq \varepsilon \tag{13}
\]

Where \( \zeta \) is either \( \psi \) or \( \Theta \), \( n \) represents the iteration number and \( \varepsilon \) is the convergence criterion. In this study, the convergence criterion is set at \( \varepsilon = 10^{-6} \). The heat transfer in term of the average Nusselt number in Eq. (12) was calculated numerically using the trapezoidal integration rule.

3. Results and Discussion

Figure 2 depicts the effect of the inclination angle of a parallelogram on the contour of temperature (\( \Theta \)) and stream function (\( \Psi \)). Note that \( \theta_s = 0^\circ \) leads to a square shape and is considered as a special parallelogram. The fluid motion as shown in the figure is described as follows. Since the temperature of the left wall is higher than that of the fluid inside the enclosure, the wall transmits heat to the fluid and raises the temperature of fluid particles adjoining the left wall. When the temperature rises, the fluid starts moving from the left wall (hot) to the right wall (cold) and falling along the cold wall, then rising again at the hot wall, creating a clockwise rotating cell inside the enclosure. The cell patterns inside the enclosure follow the parallelogrammic types due to different values of the inclination angle. Boundary layer was well developed near hot and cold wall for all the parallelogrammic types. This is due to strong buoyancy force and the fluid is less viscous at a high Rayleigh number.

Figure 3 shows that the heat transfer performance for a wide range of inclination angles. It shows that as the value of magnetic field \( Ha \) is made larger, the strength of the heat transfer is progressively suppressed. This behavior is due to the retarding effect of the electromagnetic body drags in the Lorentz force. The suppression of the heat transfer rate is more pronounced at the square (\( \theta_s = 0^\circ \)) geometry than the parallelogrammic geometry (\( \theta_s \neq 0^\circ \)). It is also observed that by tuning the inclination, the heat transfer performance is significantly decreased.
Figure 2. Contour of Temperature, $\Theta$ (Left) and Stream Function, $\Psi$ (Right) for $Ra = 500$, $Ha = 0.5$; (a) $\theta_s = -30^\circ$, (b) $\theta_s = 0^\circ$, (c) $\theta_s = 30^\circ$

Figure 3. Variation Average Nusselt Number with the Inclination Angle

4. Conclusion

The problem of natural convection flow in a parallelogrammic region filled with an electrically conducting fluid-saturated porous medium has been studied numerically. We can conclude that the convection modes within the enclosure depend upon the Rayleigh number and the inclination angle. As the value of magnetic field is made larger, the strength of the heat transfer is progressively suppressed. Tuning the inclination angle decreases the heat transfer performance. A detailed discussion on a more general configuration will constitute the subject of our subsequent investigation to treat more complex problems, such as time-dependent flows.

References