A New Method for Simulating Power Flow Density Focused by a Silicon Lens Antenna Irradiated with Linearly Polarized THz Wave

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Abstract

A terahertz system uses dielectric lens antennas for focusing and collimating beams of terahertz wave radiation. Linearly polarized terahertz wave radiation has been widely applied in the terahertz system. Therefore, an accurate method for analyzing the power flow density in the dielectric lens antenna irradiated with the linearly polarized terahertz wave radiation is important to design the terahertz systems. In optics, ray-tracing method has been used to calculate the power flow density by a number density of rays. In this study, we propose a method of ray-tracing combined with Fresnel’s transmission, including transmittance and polarization of the terahertz wave radiation to calculate power flow density in a Silicon lens antenna. We compare power flow density calculated by the proposed method with the regular ray-tracing method. When the Silicon lens antenna is irradiated with linearly polarized terahertz wave radiation, the proposed method calculates the power flow density more accurately than the regular ray-tracing.

Keywords: Fresnel’s transmission, ray-tracing, silicon lens antenna

1. Introduction

Terahertz (THz) wave radiation, which lies between microwave and infrared, has some attractive qualities, for example it can yield extremely high-resolution images, move vast amounts of data quickly, has a non-ionizing feature, and stimulate molecular and electronic motions in many materials [1]. These special characteristics have attracted many researchers to explore potentials of the THz wave and to develop the THz technology. Various studies on the applications of THz technology have been reported, such as non-destructive inspection of concealed weapons [2], food examination [3], pharmaceutical examination [4], and analysis of DNA molecules [5].

Within developments of the THz technology, source and detector performance, speed of measurement and cost, lay some issues for extending the utilization of THz waves [6]. Among these issues, we propose a way to improve THz source and detector performance through...
a dielectric lens antenna for focusing and collimating beams of THz wave radiation. The use of an dielectric lens antenna allows free-space propagation of THz waves [7]. If there is no lens attached to the back of a THz source (a THz antenna), most of the generated radiation remains trapped in the THz source substrate because of total internal reflections [8].

An extended hemispherical dielectric lens is practical, since the dielectric lens is coupled to a quasi-optical system by simply adjusting the extension length behind the hemispherical [9]. When the dielectric lens is irradiated with THz wave radiation, the THz wave radiation is focused inside the dielectric lens antenna after passing through the hemisphere boundary. The highest value of power flow density distribution is located around the focus in the extended length. If a THz detector is placed around the focus in the extension, the detector detects the highest power flow density. As a result, the detector coupled to the dielectric lens has better detection than without the dielectric lens.

A Silicon lens (Si-lens) antenna, as one of the dielectric lens antennas, is used widely in THz systems, for example to improve the coupling efficiency in a THz quantum cascade laser (QCL) [10], photoconductive antenna (PCA) [11], a hot electron bolometer (HEB) [12] and an antenna coupled detector [13]. A number of groups have reported the development of the Si-lens antenna [14,8,15,16]. Therefore, a reliable method to precisely evaluate power flow density distribution in the Si-lens antenna irradiated with linearly polarized THz wave radiation is important to design THz systems.

At a high frequency, the transmission of a spherical or plane wave through an arbitrarily curved dielectric interface can be solved by the geometrical optics theory [17]. Ray-tracing method based on the geometrical optics theory is used to analyze beams of terahertz radiation in a quasi-optical system through a number density of rays [14]. In optics, ray-tracing is widely used to design lenses and to analyze the optical system. Figure 1 shows the ray-tracing of several rays on a Silicon lens simulated by an optical ray-tracing software (Zemax). The rays are collected into focus in the extension of the hemisphere after passing through the hemisphere boundary.

When we calculate the power-flow density inside the Si-lens antenna for linearly polarized THz wave radiation, we should consider the THz wave polarization and transmittance, which are not considered by the ray-tracing method. Polarization is defined as the direction of electromagnetic wave’s oscillation, while transmittance is the ratio of the power passed through the boundary to the power of incidence. Based on Fresnel’s equation of transmission, there are two components of transmittance, i.e. parallel ($T_P$) and perpendicular ($T_S$) components.

When a ray comes from medium 1 ($n_1$) to medium 2 ($n_2$) with different refractive indexes, the two components are determined by the plane of incidence and the direction of the THz wave polarizations, as shown in Figure 2. The angle of incidence and the plane of incidence for every ray passed through the hemisphere boundary is different from the transmittances of other rays. This condition leads to different transmittances of every ray that passes through the boundary.
Analysis of complex electrically large structures using the method of moments (MoM), the finite element method (FEM), and the finite difference time domain (FDTD) method can become prohibitive due to the need for large computational resources [18]. Asymptotic methods, such as geometrical optics (GO) [17] and physical optics [19], on the other hand require much less computational resources and are effective in modelling electrically large structures but only for far-field calculations at high frequencies.

Spectral Domain Ray tracing (SRT) based on the spectral theory of diffraction (STD) has been proposed to improve the limitation of the geometrical optics at lower frequencies [18]. The SRT represents the spectral samples of the plane waves as ray tubes that leave the source plane and reach an observation point. The geometric theory of diffraction (GTD) is applied to these rays, where they undergo reflections, refractions, and diffractions. The SRT method has proven to be as accurate as FEM and yet as computationally fast [18,20].

In this study, we propose a method for evaluating power-flow density distribution focused by a Si-lens antenna irradiated with linearly polarized THz wave radiation at frequency one THz. The proposed method combines the ray-tracing method and Fresnel’s transmission to include the linearly polarized THz wave parameters. The evaluation of the power flow density distribution around the focus is necessary for designing the dielectric lens, which is used to improve the THz wave detector. We consider high-resistivity silicon for the dielectric lens antenna, which has a very low dispersion of refractive index and low absorption in the range of 0.3-2.5 THz [21].

2. Experiment

A Model to Evaluate the Power-flow Density. Figure 3 shows a scheme of a ray of incidence, and a ray of refraction (transmission). The direction of the ray of incidence is the same as the z-axis direction. The Si-lens antenna consists of a hemisphere and a cylinder extension. The radius \( R \) of the hemisphere and the length \( d \) of the cylinder extension are two millimeters and one millimeter, respectively. The dielectric constant \( (\varepsilon_r) \) of the Si-lens antenna is 11.7 at one THz. The normal vector (dash line) of the hemisphere boundary is obtained by making a line from the hemisphere center to the observed boundary.

When uniform parallel rays travel to the Si-lens antenna, we calculate the angles of incidence \( (\theta_i) \) for rays that reach the hemispherical boundary as defined in Equation 1. The ray of incidence, the normal vector of the boundary, and the ray of refraction (transmission) make a plane of incidence, as shown in Figure 2. By following Snell’s law of refraction as defined in

Equation 2, the refractive index of air \( (n_a = 1) \) and the refractive index of the Si-lens material \( (n_s = \sqrt{\varepsilon_r}) \) are used to calculate the angles of refraction \( (\theta_r) \). The angles of refraction \( (\theta_r) \) bend the transmitted rays after passing through the hemisphere boundary and collected at around a focus after passing through the hemisphere boundary.

By following Fresnel’s law of transmission as defined in Equations 3 and 4, we calculate transmittances for all transmitted rays by taking into account the angles of incidence \( (\theta_i) \), the refractive indices \( (n_a, n_s) \), the angles of refraction \( (\theta_r) \), the planes of incidence, and the THz wave polarization. Figure 4 shows a vector of the two transmittance components, which is determined by the direction of polarization and the plane of incidence (dash line). In this study, we consider the direction of polarization parallel to the x-axis. The parallel component is the component whose vector is parallel to the plane of incidence. The perpendicular component is the component whose vector is perpendicular to the plane of incidence. Equation 5 is used to calculate the transmittance deduced from the two components. Angle \( \theta_r \) calculated by Equation 6 is a tangent function from the observed ray on x-y plane.

\[
\theta_i = \sin^{-1} \left( \frac{\varepsilon_r}{n_a} \right) \tag{1}
\]

\[
\theta_r = \sin^{-1} \left( \frac{n_a}{n_s} \sin (\theta_i) \right) \tag{2}
\]

\[
T_p = 1 - \left| \frac{\sin(n_a n_s \sin(\theta_i))}{\sin(\theta_i \varepsilon_r n_s)} \right|^2 \tag{3}
\]
The transmitted rays are traced inside the Si-lens antenna. When a transmitted ray reaches z-axis, as shown in Figure 5, it makes a triangle of ABC from the hemisphere boundary to z-axis. Angle of $\theta_2$ can be calculated with Equation 7 as a function of the angle of incidence ($\theta_1$) and the angle of refraction ($\theta_2$). By considering the triangle OBC and OC equal to the radius of the Si-lens antenna, the length of BC can be calculated by Equation 8. After the angle of $\theta_2$ and the length of BC have been calculated with Equation 7 and 8, respectively, Equation 9 is used to calculate the length of AB. By using Equation 10, we calculate the length of OA to get the distance from the hemisphere center or the origin of the Cartesian coordinates to point A. Equation 11 is used to calculate the distance of the transmitted rays from the z-axis. After obtaining the length of $y'$ from Equation 11 and the angle of $\theta$ from Equation 6 for all transmitted rays, we can identify the position of the transmitted rays on the x-y planes as polar coordinates ($y$, $\theta$).

\[
\theta_2 = \theta_1 - \theta_2
\]

(7)

\[
BC = R \sin(\theta_1)
\]

(8)

\[
AB = \frac{BC}{\tan(\theta_1)}
\]

(9)

\[
OA = (R - \frac{R \cos(\theta_1)}{\tan(\theta_1)}) + \left(\frac{AB}{\tan(\theta_1)}\right)
\]

(10)

\[
y' = (OA - Z_m) \tan(\theta_1)
\]

(11)

The power-flow density for a transmitted ray on a plane perpendicular to the z-axis is calculated by taking into account the Fresnel’s transmission and the value of the power-flow density in a unit area (VA/m²) of the THz wave radiation. We determine the power-flow density of the rays of incidence ($R_2$), therefore the power-flow density of the transmitted ray ($R_2$) is the power-flow density of the ray of incidence ($R_2$) multiplied by the Fresnel’s transmission ($T$), as in Equation 12.

\[
R_2 = R_2 T
\]

(12)

After the transmitted ray distributions on the x-y planes in the Si-lens antenna have been calculated with Equations 6 and 11, we calculate the power-flow density distribution by summing the power-flow density for all transmitted rays ($R_2$), which are in the same square. Figure 6 shows square meshes used to calculate the power-flow density distribution. Every square unit has length of delta (\(\Delta\)). As the power-flow density calculation is done on the x-y plane, a two-dimensional matrix (m, n) is used to identify the distribution of the power-flow density.

\[
\text{Figure 5. Scheme of a Ray of Incidence and a Ray of Transmission Around the Boundary of Air to the Si-lens Antenna}
\]

\[
\text{Figure 4. E-field Vector ( ) of an Incoming Ray on the x-y Plane and the two Transmittant Components of Parallel ( ) and Perpendicular ( ) to the Plane of Incidence Around the Hemisphere Boundary}
\]

\[
\text{Figure 6. Square Meshes Used to Calculate the Power-flow Density Distribution on x-y Plane}
\]
Analysis of the power-flow density focused by the Si-lens antenna is done by observing the power-flow density distribution of several x-y planes from the center (z-axis) to the edge of the Si-lens antenna. After the rays are passed through the hemisphere boundary, the power-flow density on the x-y planes is formed in circles. The radii of the circles are used to find differences of the power-flow density among the observed planes. The radii of circles as a function of the z-axis (z > 0) will show the focusing pattern in the cylinder extension of the Si-lens antenna. We calculated the radii for different percentages of the total transmitted power in the Si-lens antenna calculated from the center of the circles (z-axis) to the edge of the Si-lens antenna to show the effect of the Fresnel’s transmission. The radii for the different percentages are also used to show the concentration of the power-flow density. To clarify the results of our proposed method, we used an optical software simulator of Zemax, which is based on regular ray-tracing.

3. Results and Discussion

From our calculation, the parallel rays of THz beams are irradiated into the Si-lens antenna. Our purpose is to provide a method for evaluating power-flow density distribution with linearly polarized THz wave radiation at frequency one THz. The physical parameters of the Si-lens antenna have been discussed in Section 2. We consider high-resistivity silicon for the dielectric lens antenna, which has a very low dispersion of refractive index and low absorption in the range of 0.3-2.5 THz [21].

Figure 7 (a) and (b) show transmittance on the x-axis (y=0) and y-axis (x=0), respectively, as the linearly polarized THz wave radiation passes through the hemisphere boundary of the Si-lens antenna. Figure 7 (a) shows the maximum transmittance appears near the edge of the Si-lens antenna, which is Brewster’s angle at around 76.9°. The power of the THz wave irradiated to the Si-lens antenna is completely transmitted (100% transmittance) at the Brewster’s angle. The Brewster’s angle is obtained because the transmittance on the x-axis consists of parallel component and zero perpendicular components. When the transmittance consists of perpendicular component and zero parallel component, Figure 7 (b) shows the maximum transmittance on the y-axis appears at the center and decreases monotonously to the edge. In our previous work [22], we have shown the transmittance distribution just after passing from air to the Si-lens antenna for the linearly polarized THz wave at frequency one THz. The transmitted power passed through the Si-lens antenna is about 68.3%. This value is smaller than the transmitted power of the normal incidence passed through the Silicon material, which is around 70.0%.

Table 1 is a summary of the position of the focuses at z-axis and the length of the radii for six different percentages (50%, 60%, 70%, 80%, 90% and 100%). The radii around the focus, which is the location of the highest value of the power-flow density distribution, decreases once the percentage increases. The changes of the radii are affected by spherical aberration. The spherical aberration occurs because the Si-lens antenna has a hemisphere boundary, which is a large relative aperture. Therefore, the focus is no longer a single point. When we design a bolometer THz wave detector or a THz transceiver antenna, which will be coupled to the Si-lens antenna, the information as shown in Table 1 is useful.
Figure 8. The Radii of Circles for 50%, 70% and 90% of the Transmitted Power in the Cylindrical Extension Calculated by the Ray-tracing with Fresnel’s Transmission and the Regular Ray-tracing Software (Zemax)

Table 1. The Location of the Focus on the z-axis and Length of the Radii for Some Percentages of the Transmitted Power Calculated by the Proposed Method

<table>
<thead>
<tr>
<th>%</th>
<th>z-axis (mm)</th>
<th>Radius (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.78</td>
<td>12.99</td>
</tr>
<tr>
<td>60</td>
<td>0.76</td>
<td>14.18</td>
</tr>
<tr>
<td>70</td>
<td>0.74</td>
<td>21.45</td>
</tr>
<tr>
<td>80</td>
<td>0.72</td>
<td>29.99</td>
</tr>
<tr>
<td>90</td>
<td>0.70</td>
<td>39.87</td>
</tr>
<tr>
<td>100</td>
<td>0.64</td>
<td>93.41</td>
</tr>
</tbody>
</table>

useful to find the proper size and absorbing pattern (or radiation pattern of an THz antenna) of the THz wave detector or the THz transceiver. As high-resistivity Silicon has a very low dispersion of refractive index and low absorption in the frequency range of 0.3-2.5 THz [16], our proposed method is applicable for calculating power-flow density focused by the Si-lens antenna in the wide range of THz region.

4. Conclusions

The power-flow density focused by a Silicon lens antenna for linearly polarized THz wave radiation has been calculated by ray-tracing method combined with Fresnel’s transmission. By including transmittance and polarization, the proposed method calculates the power-flow density more accurately than the results calculated with the regular ray-tracing. The proposed method is expected to contribute to designing dielectric lens antenna to develop the compact THz system.

References