

# Residual Strength Analysis of Asymmetrically Damaged Ship Hull Girder using Beam Finite Element Method

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## Abstract

The objective of the present study is to analyze the residual strength of asymmetrically damaged ship hull girder under longitudinal bending. Beam Finite Element Method is used for the assessment of the residual strength of two single hull bulk carriers (Ship B1 and Ship B4) and a three-cargo-hold model of a single-side Panamax Bulk Carrier in hogging and sagging conditions. The Smith's method is adopted and implemented into Beam Finite Element Method. An efficient solution procedure is applied; i.e. by assuming the cross section remains plane, the vertical bending moment is applied to the cross section and three-cargo-hold model. As a fundamental case, the damage is simply created by removing the elements from the cross section, neglecting any welding residual stress and initial imperfection. Also no crack extension is considered. The result obtained by Beam Finite Element Method so-called Beam-HULLST is compared to the progressive collapse analysis obtained by HULLST for the validation of the present work. Then, for the three-hold-model, the Beam-HULLST is used to investigate the effect of the rotation of the neutral axis both intact and damage condition taking the one and five frame spaces into account.

## Abstrak

**Analisis Kekuatan Sisa Penumpu Lambung Kapal Asimetris Pasca Rusak dengan Metode *Beam Finite Element*.** Tujuan dari studi ini adalah untuk menganalisis kekuatan sisa dari penumpu lambung kapal rusak tidak simetris dalam pengaruh lentur memanjang. Metode *Beam Finite Element* diadopsi untuk pengujian dari kekuatan sisa dari dua kapal bulk carrier (Ship B1 dan Ship B4) dan sebuah model tiga-ruang-muat dari kapal *bulk carrier* dengan tipe Panamax berlambung tunggal pada kondisi *hogging* dan *sagging*. Suatu prosedur penyelesaian yang efisien dengan kata lain lambung kapal diasumsikan tetap pada bidang, momen lentur vertikal bekerja pada penampang dan model tiga-ruang-muat. Untuk kasus kerusakan, bagian yang rusak dibuat sederhana dengan menghilangkan elemen-elemen dari penampang, tegangan sisa pengelasan, dan ketidaksempurnaan awal diabaikan. Tidak ada perpanjangan retak yang dipertimbangkan. Hasil yang diperoleh dengan menggunakan metode *Beam Finite Element* disebut Beam-HULLST dibandingkan dengan analisis *progressive collapse* yang diperoleh dengan menggunakan HULLST untuk validasi dari metode yang digunakan. Kemudian, pada model tiga-ruang-muat, digunakan Beam-HULLST untuk menginvestigasi pengaruh rotasi sumbu netral pada kondisi *intact* dan *damage* dengan mempertimbangkan satu dan lima jarak gading.

*Keywords: beam finite element, bulk carrier, damage, hull girder, residual strength*

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## 1. Introduction

To avoid a collapse of the ship hull under normal circumstances, design rules given by classification societies define a maximum stress level, which should not be exceeded under the prescribed extreme loading condition or alternately a minimum elastic section modulus required [1-4]. These have proven to be effective for intact ships in normal seas and loading conditions. However, their applicability to assess the survivability of ships in accidental situation, e.g. collision

or grounding damage, is uncertain due to the interacting effects of local yielding, buckling or rupture as well as due to the loading on the hull. In this regard, the ultimate strength is a better basis for safety assessment as well as design, because it can define the true ultimate limit state.

The progressive collapse behavior of an asymmetrically damaged cross section under longitudinal bending moment corresponds to the collapse behavior of a hull girder having damage of large length and subjected to uniform bending moment at the damaged part [5]. In

more general case, however, the damage length is limited and thus the effect of the rotation of the neutral axis due to an asymmetric damage is confined and constrained by the intact part [6-7]. In addition, the loading condition may change both in magnitude and distribution after the damage, especially when heeled angle increases due to flooding. One strategy to consider such a limited damaged length and the change in the external load distributions on the hull girder is to idealize the whole or part of hull girder using beam finite elements, and the damage effect is introduced to some particular elements. For the rapid judgment of a survivability of damaged hull girder in emergency, a simplified and efficient approach such as a beam model is required [8-9].

From this viewpoint, a method of the progressive collapse analysis of a ship hull girder with asymmetric damages is developed using the beam finite element and introducing the Smith approach to each element [10], and the program code Beam-HULLST is developed. The constraining effect of the intact parts on the damaged part where the neutral axis rotates and the effect of localization of the plastic deformation at the damaged part on the collapse behavior of a whole ship are examined. Therefore, the objective of the present study is to analyze the ultimate strength of asymmetrically damaged ship hull girder strength under longitudinal bending.

**Fundamentals of thin-walled beam.** It has been proven practice to use simple beam theory to analyze the progressive collapse behavior of ship hull girder under longitudinal bending. Many experiments have confirmed that the bending behavior of ships agrees quite well with beam theory. The hull girder of cross section represents the bending strength of the primary hull structure. This means that the calculation of hull girder cross-section is very important for the ship design. Structural members that continue in longitudinal direction are included in the calculation of cross section. The members are divided into plates and stiffeners with attached plating. Smith's method has been widely employed to handle this procedure.

The formulation in Beam-HULLST is based on the thin-walled beam theory. Here, the fundamental theory of thin-walled beam element includes the torsion effect as a general case. The coordinate system is defined as shown in Figure 1. The x- and y-axes are defined on the beam cross section and the z-axis is parallel to the beam axis. The origin of the coordinate system is located at the gravity center of the cross section. The s-coordinate is defined along the mid-thickness line.

Assuming that the cross section remains undistorted during deformation, the displacement U, V and W in the x, y, and z directions at the coordinate (x,y,z) can be expressed as

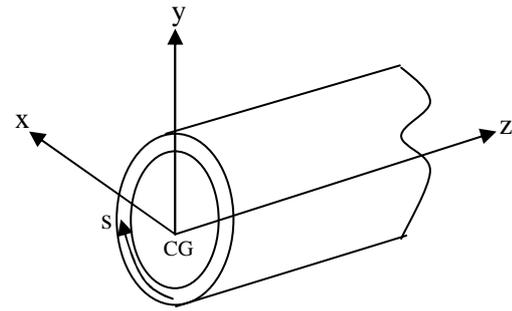


Figure 1. Coordinate of Thin-Walled Beam

$$U(x, y, z) = u_s(z) - (y - y_s)\theta(z) \tag{1}$$

$$V(x, y, z) = v_s(z) + (x - x_s)\theta(z) \tag{2}$$

$$W(x, y, z) = w(z) - xu'_s(z) - yv'_s(z) + \omega_{ns}(x, y)\theta'(z) \tag{3}$$

Where  $u_s$  and  $v_s$  are the displacements at the shear center in x and y direction and  $w$  is the displacement at the gravity center in z direction.  $\theta$  is the rotation angle about the shear center.  $x_s$  and  $y_s$  are x and y coordinates of the shear center.  $\omega_{ns}$  is the warping function about the shear center. A prime (') denotes differentiation with respect to the z-coordinate. The strains based on the displacements of Eq.(1) and Eq.(2) can be expressed as

$$\epsilon_z = w' - xu''_s - yv''_s + \omega''_{ns}\theta'' \tag{4}$$

$$\begin{aligned} \gamma_{sz} &= \gamma_{xz} \frac{\partial x}{\partial s} + \gamma_{yz} \frac{\partial y}{\partial s} = \left\{ \frac{\partial \omega_{ns}}{\partial s} - (y - y_s) \frac{\partial x}{\partial s} \right. \\ &= \left. + (x - x_s) \frac{\partial y}{\partial s} \right\} \theta' \end{aligned} \tag{5}$$

Where  $\epsilon_z$  is the strain in the z-axis direction.  $\gamma_{sz}$  is the shear strain in the s-z plane. The stress and strain relationship can be expressed as

$$\begin{pmatrix} \sigma_z \\ \tau_{sz} \end{pmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{pmatrix} \epsilon_z \\ \gamma_{sz} \end{pmatrix} \tag{6}$$

Where  $\sigma_z$  is the axial stress,  $\tau_{sz}$  is the shear stress in the s-z plane, and  $d_{ij}$  gives a stress-strain relationship.

In the present work, the ship cross section is modeled by thin-walled beam. Hence,  $d_{ij}$  corresponds to the stiffness of segmented members, which consist of plate and stiffened plate where it also depends on the yielding and buckling. For the case of finite element formulation, a beam element  $ij$  is considered, and it is divided in z direction as shown in Figure 2. The length of the element

is denoted by.  $\{u_s\}$  is the nodal displacement vector at the shear center, consisting of the translation in x direction and its derivative with respect to z (denote by ') as

$$\{u_s\}^T = [u_{si}, u_{si}', u_{sj}, u_{sj}'] \quad (7)$$

where the subscript  $i$  means the node  $i$ . Similarly, other nodal displacements can be expressed as

$$\{v_s\}^T = [v_{si}, v_{si}', v_{sj}, v_{sj}'] \quad (8)$$

$$\{\theta_s\}^T = [\theta_i, \theta_i', \theta_j, \theta_j'] \quad (9)$$

$$\{w\}^T = [w_i, w_j] \quad (10)$$

$$\{d\}^T = [\{u_s\}, \{v_s\}, \{\theta\}, \{w\}] \quad (11)$$

Where  $\{v_s\}$  is the nodal displacement vector at shear center in y direction,  $\{\theta\}$  is the torsion angle about shear center and torsion rate, and  $\{w\}$  is the axial displacement at the gravity center. Correspondingly, the nodal forces are defined as

$$\{F_u\}^T = [F_{xi}, M_{yi}, F_{xj}, M_{yj}] \quad (12)$$

$$\{F_v\}^T = [F_{yi}, M_{xi}, F_{yj}, M_{xj}] \quad (13)$$

$$\{F_\theta\}^T = [T_i, B_i, T_j, B_j] \quad (14)$$

$$\{F_w\}^T = [F_{zi}, F_{zj}] \quad (15)$$

$$\{F\}^T = [\{F_u\}, \{F_v\}, \{F_\theta\}, \{F_w\}] \quad (16)$$

Where  $\{F_u\}$  and  $\{F_v\}$  are the shear forces and bending moments,  $\{F_\theta\}$  the torsion moment and bi-moment about

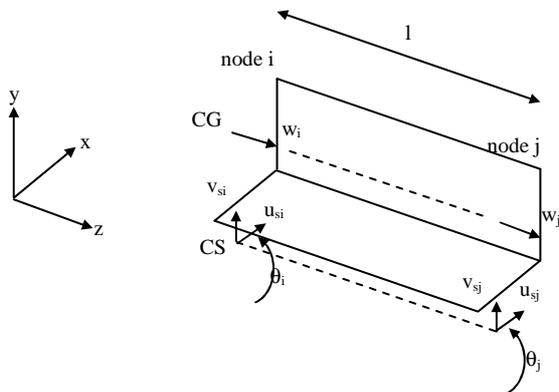


Figure 2. Beam Element ij

the axis at the shear center, and  $\{F_w\}$  the axial force. The axial displacement  $w(z)$  is interpolated linearly within the element and the horizontal and vertical deflections  $u_s(z)$ ,  $v_s(z)$ , and torsion angle  $\theta(z)$  by cubic polynomials.

Using the nodal displacement and nodal coordinates, the displacement functions can be expressed as a function of nodal displacement in the form

$$\begin{aligned} u_s(z) &= [A_c(z)]\{u_s\} \\ v_s(z) &= [A_c(z)]\{v_s\} \\ \theta(z) &= [A_c(z)]\{\theta\} \\ w(z) &= [A_L(z)]\{w\} \end{aligned} \quad (17)$$

where

$$[A_c(z)] = [1 \quad z \quad z^2 \quad z^3] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{l^2} & \frac{2}{l} & \frac{3}{l^2} & \frac{2}{l} \\ \frac{2}{l^3} & \frac{1}{l^2} & -\frac{2}{l^3} & \frac{1}{l^2} \end{bmatrix}$$

$$[A_L(z)] = [1 \quad z] \begin{bmatrix} 1 & 0 \\ -\frac{1}{l} & \frac{1}{l} \end{bmatrix}$$

Substituting Eq.(17) into Eq.(4) and Eq.(5), the axial and shear strain can be expressed as

$$\begin{pmatrix} \varepsilon_z \\ \gamma_{sz} \end{pmatrix} = \begin{bmatrix} [B_1] & -x[B_2] & -y[B_3] & \omega_{ns}[B_2] \\ 0 & 0 & 0 & g(s)[B_3] \end{bmatrix} \{d\} \quad (18)$$

Where

$$[B_1] = \frac{d}{dz} [A_L]$$

$$[B_2] = \frac{d^2}{dz^2} [A_c]$$

$$[B_3] = \frac{d}{dz} [A_c]$$

$$g(s) = \frac{\partial \omega_{ns}}{\partial s} - (y - y_s) \frac{\partial x}{\partial s} + (x - x_s) \frac{\partial y}{\partial s}$$

Applying the principle of virtual work to the stress and strain increment, the incremental form of the stiffness equation is derived in the form

$$\{\Delta F\} = [K] \{\Delta d\} \quad (19)$$

where the stiffness equation  $[K]$  is given by

$$[K] = \int_V \begin{bmatrix} d_{11}[B_1] & -xd_{11}[B_{12}] & -yd_{11}[B_{12}] & \omega_{ns}d_{11}[B_{12}] \\ -xd_{11}[B_{21}] & x^2d_{11}[B_2] & xyd_{11}[B_2] & -x\omega_{ns}d_{11}[B_2] \\ -yd_{11}[B_{21}] & xyd_{11}[B_2] & y^2d_{11}[B_2] & -y\omega_{ns}d_{11}[B_2] \\ \alpha d_{11}[B_{21}] & -x\omega_{ns}d_{11}[B_2] & -y\omega_{ns}d_{11}[B_2] & \omega_{ns}^2d_{11}[B_2] + g^2d_{22}[B_3] \end{bmatrix} \quad (20)$$

In case of the elastic cross section with uniform material properties, the stress-strain relationship of Eq. (6) at the arbitrary points given by  $d_{11} = E$ ,  $d_{22} = G$ ,  $d_{12} = d_{21} = 0$ , where  $E$  is young's modulus and  $G$  is shear modulus. In the progressive collapse analysis,  $d_{ij}$  must be changed considering buckling and yielding. In this study, buckling and yielding of stiffened panel under axial load is predominantly considered in the residual strength assessment.

As a most simple approach, the interactive term is ignored, i.e.  $d_{ij} = 0$  ( $i \neq j$ ), and the axial stiffness  $d_{11}$  is calculated by HULLST, and the shear stiffness is set as  $d_{22} = G$  before the ultimate strength and  $d_{22} = 0$  beyond the ultimate strength. More rational formulation of  $d_{ij}$  is needed in the future including the effect of shear failure.

## 2. Methods

Progressive collapse analysis of hull girders with collision damage was performed using the beam finite elements in which the Smith's method is implemented for the calculation of the axial stress-strain relationship of the plate and stiffened panel members.

The procedure of the progressive collapse analysis using thin-walled elastic beams in this study is summarized as follows: a) Ship hull girder cross section was idealized by the beam elements; b) Following the Smith's method, the cross section of each beam element was divided into plate and stiffened plate as shown in Figure 3; c) Elastic stiffness matrix of beam elements was calculated using Eq.(20); d) Progressive collapse analysis was performed by applying prescribed force or curvature at the beam nodes.

Two types of beam models were employed as shown in Figure 4. One is the one frame-space model and the other the five frame-space model. The damage length was assumed to be one frame-space length in both models. Seventy percent damage was assumed in one side of the cross section. The analysis of the one-frame space model corresponded to the progressive collapse analysis of the damaged cross section obtained by HULLST. In this case, two types of single hull bulk carrier namely ship B1 and ships B4 were used. Dimensions and design criterion are summarized in Table 1.

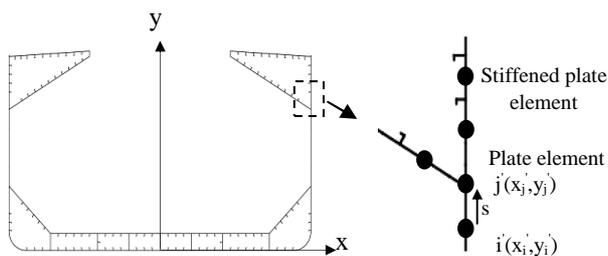
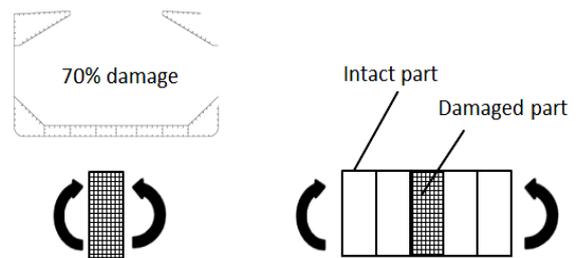


Figure 3. Element Division on the Cross Section

On the other hand, the five-frame space model included the damaged part partially, and thus the constraining effect of the intact part on the deformation of the damaged part due to the rotation of the neutral axis could be considered.

The boundary condition for the beam models is shown in Figure 5. The forced rotation angles about horizontal axis were applied at the both-end nodes in the opposite directions. To allow for the shift of the neutral axis during the progressive collapse, the longitudinal translation at one end was fixed and the other end set free under the condition of zero axial loads.

In addition, the rotation about the vertical axis as well as about the horizontal axis was allowed at both ends to consider an occurrence of the horizontal curvature under vertical bending moment due to the rotation of the neutral axis the damaged part (Case 1). For the one frame space model, the analysis with the rotation about vertical axis fixed (Case 2) was also performed for comparison purpose.



(a) 1 frame space model (b) 5 frame space model

Figure 4. Beam-HULLST Model for Analysis

Table 1. Ship Dimensions

Ship	B1	B4
L (m)	217	219
B (m)	32,236	32,240
D (m)	18,3	19,9
Design criterion	Pre-IACS UR	IACS CSR-B

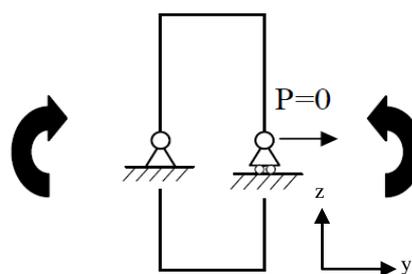
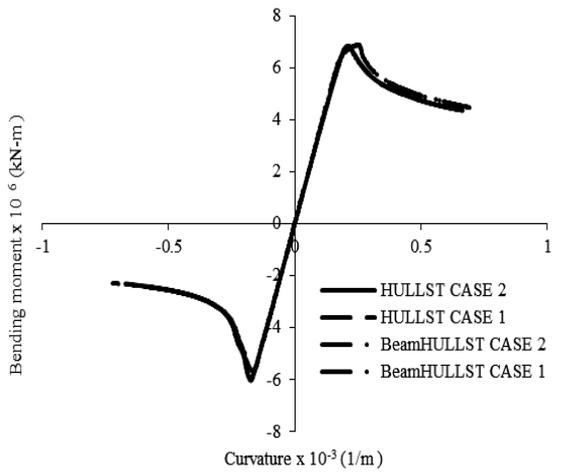


Figure 5. Boundary Condition

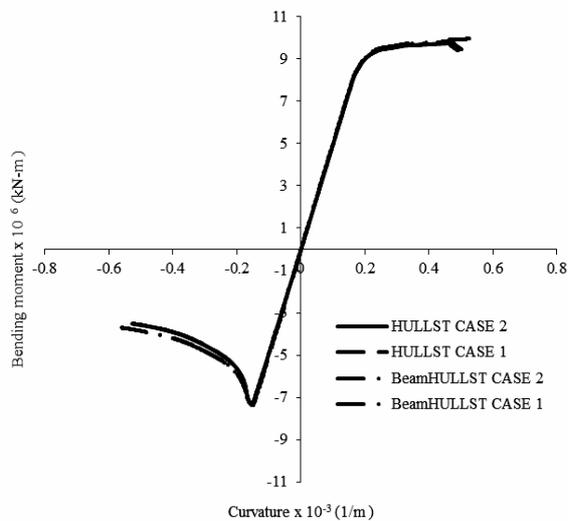
### 3. Results and Discussion

Two single hull bulk carriers, B1 and B4 were taken as the subject ships as shown in Table 1. The damage was located at asymmetric position on the side shell of the hull girder cross section. The vertical damage extent was taken for investigation. The ultimate strength was 70% of ship depth. The horizontal damage extent was taken B/16, and it was kept constant for all damage cases.

The vertical bending moment and vertical curvature relationships obtained by HULLST and those by Beam-HULLST with one frame space model are compared in Figure 6. The residual of the ultimate strength for ship B1 and ship B4 are described in Figure 6(a) and (b), respectively. As shown, two analysis methods generated



(a) Ship B1



(b) Ship B4

Figure 6. Moment-Curvature Relationship for Intact of Ship B1 (a) and Ship B4 (b)

almost identical results. The reasonable agreement between two programs was obtained. The difference of the residual hull girder strength between ship B1 and ship B4 might be caused by the ratio breadth (B) and depth (D) of the models. The residual of the ultimate strength was obtained by two programs when the rotation of the neutral axis constrained (case 2) was larger compared when it was not considered (case 1).

Figure 7 shows the bending moment–curvature relationships of a single-side Panamax-size bulk carrier obtained by the Beam-HULLST. The three-cargo-hold model of the subject ship is shown in Figure 8. The structural member dimensions were determined based on the IACS/CSR-B. Figure 7 compares the results obtained by the one frame-space model and the five frame-space model. The average curvature was calculated by dividing the relative rotation angle between both-end cross sections by the overall length of the model.

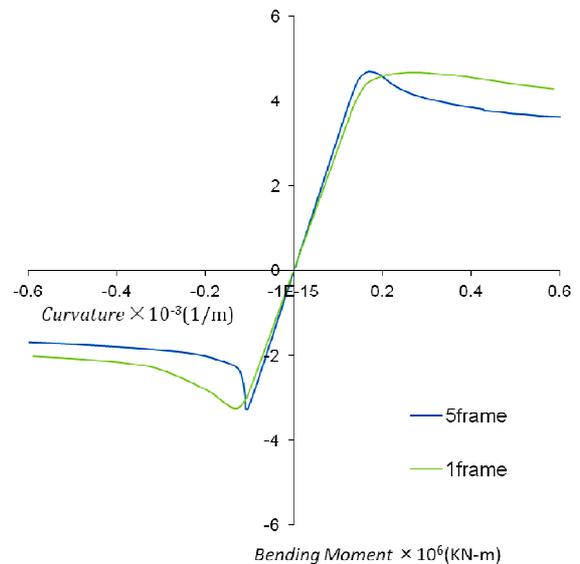


Figure 7. Bending Moment-Curvature Relationship of a Panamax-Size Bulk Carrier obtained by Beam-HULLST using One Frame-Space Model and Five Frame-Space Model.

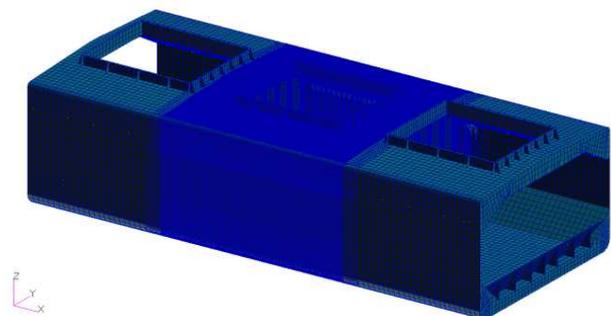


Figure 8. Three-Cargo-Hold Model of Paramax-Size Bulk Carrier

It was found that the five frame-space model consisting of one damaged frame space at the middle and the intact frame space for the rest part gave slightly larger ultimate strength than that obtained by the one frame space model. This was because the effect of the rotation of the neutral axis at the damaged part was constrained by the presence of the intact part. The reduction of the residual strength due to the rotation of the neutral axis was found to be further smaller than that specified in the IACS draft rule.

It was also found, as seen in Figure 7, that the bending moment capacity beyond the ultimate strength of the five frame-space model decreased more rapidly than that of the one-frame-space model. This was because of localization of the plastic deformation at the damaged cross section and the simultaneous unloading in the rest part of the model. As shown, the collapse of the damaged ship took place in more brittle manner (sudden drop of the capacity) than that predicted by the bending moment-curvature relationship of the cross section as obtained by HULLST.

This suggests the need for the Beam-HULLST model that can deal with the behavior of the whole hull girder.

#### 4. Conclusions

Residual strength analysis of bulk carriers with collision damages has been performed using Beam-HULLST. Thus, we can conclude as follows: a) It was confirmed that the progressive collapse behavior obtained by the Beam-HULLST using one beam element almost coincides with the result by HULLST for a cross section; b) The influence of the rotation of the neutral axis at the asymmetrically damaged cross section was reduced by the presence of the adjacent intact parts. The reduction rate of the residual hull girder strength should

be smaller than that found in the analyses for a cross section; c) The localization of the plastic deformation at the damaged part and the simultaneous unloading at the rest part of the hull girder had a significant influence on the post-ultimate strength behavior for the hull girder. The Beam-HULLST that could handle this effect was effective for the risk assessment of the damaged hull girder in the damaged condition.

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