

Discrete Power Control in Heterogeneous Networks

Yuanyuan Wang, Wibowo Hardjawana*, and Branka Vucetic

School of Electrical and Information Engineering, The University of Sydney, New South Wales 2006, Australia

*e-mail: wibowo@sydney.edu.au

Abstract

In heterogeneous networks (HetNets) where femtocell base stations (FBSs) are deployed within the radio coverage of macrocell base stations (MBSs) to increase network capacity, co-channel interference limits overall system performance with universal frequency reuse. This paper investigates new distributed downlink discrete power control scheme for FBSs in HetNets with FBSs cooperation. The objective of the proposed power control scheme is to maximize the number of simultaneous FBSs transmissions in a single transmission wireless channel where each FBS is allowed to transmit only if the signal-to-interference-noise ratio (SINR) requirements for both FBSs and MBS users are satisfied. We apply a stochastic learning automata technique to FBSs where each FBS is treated as a learning automaton and maintains a probability vector to select its discrete transmit power. During the learning process, each FBS adjusts its probability vector based on the feedback from FGW that indicates the number of FBSs transmissions that can be supported under the SINR requirement constraints of FUEs and MUEs. Simulation results show the proposed algorithm can achieve more than twice the number of simultaneous FBS transmissions achieved by existing schemes in the literature.

Abstrak

Kontrol Daya Terpisah dalam Jaringan Heterogen. Dalam jaringan heterogen (HetNets) di mana BTS *femtocell* (FBS) dikerahkan dalam cakupan radio BTS *macrocell* (MBS) untuk meningkatkan kapasitas jaringan, interferensi saluran bersama (*co-channel*) membatasi kinerja sistem secara keseluruhan dengan menggunakan kembali frekuensi universal. Makalah ini menyelidiki skema kontrol daya terpisah *downlink* yang baru didistribusikan untuk FBS di HetNets dengan kerjasama sejumlah FBS. Tujuan dari skema kontrol listrik yang diusulkan adalah untuk memaksimalkan jumlah transmisi FBS simultan dalam saluran nirkabel transmisi tunggal di mana setiap FBS diperbolehkan untuk mengirimkan hanya jika persyaratan rasio *signal-to-interference-noise* (SINR) untuk kedua FBS dan pengguna MBS terpenuhi. Kami menerapkan teknik *stochastic learning automata* untuk FBS, di mana setiap FBS diperlakukan sebagai *learning automaton* dan mempertahankan vektor probabilitas untuk memilih daya pancar terpisah. Selama proses belajar, masing-masing FBS menyesuaikan vektor probabilitas berdasarkan umpan balik dari FGW yang menunjukkan jumlah transmisi FBS yang dapat didukung di bawah kendala persyaratan SINR dari FUE dan MUE. Hasil simulasi menunjukkan algoritma yang diusulkan dapat mencapai lebih dari dua kali jumlah transmisi FBS simultan yang dicapai berbagai skema lain yang ada dalam literatur.

Keywords: heterogeneous networks, wireless cellular networks

1. Introduction

Recent studies have shown that in cellular networks about 60% of all voice calls and 90% of all data services take place in indoor environments [1]. However, due to poor indoor propagation conditions caused by high wall penetration loss, it is difficult for conventional macrocell base stations (MBS) to provide high-quality data transmissions for customers in indoor environment. To address this problem, femtocell base stations (FBS) [2] have been deployed in indoor environments.

FBSs are low-power, low-cost access points. They are installed at the indoor premises and connect to a mobile operator's network via residential digital subscriber line (DSL) or cable broadband connections. By using FBSs, indoor users can receive better signal-to-noise ratios due to the close proximity between transmitters and receivers. FBSs are usually installed by customers to increase their own data transmission qualities. Thus, they are more likely to operate in closed access (CA) mode where only authorized users can have access to them [3]. As a result, it is very likely that FBSs will have overlapping

radio coverage within existing MBSs. Given the scarcity of the frequency bands, it is preferable for FBSs and MBSs to use the same spectrum band. This results in two types of interference: the cross-tier interference between femtocell and macrocell; and intra-tier interference between neighboring femtocells. In the literature, this type of networks is referred to as a heterogeneous network (HetNet) [4]. These interferences will greatly degrade the network performance when FBSs are densely deployed [5].

Consequently, interference mitigation is a major challenge for HetNets. In HetNet where base stations use the same spectrum band, power control has been proved to be efficient to mitigate co-channel interferences [6-9]. Downlink adaptive power level setting (APLS) schemes are proposed in [6,7] where each FBS adjusts its transmit power so that the MBS' scheduled users (MUEs) can meet its minimum received signal-to-interference-noise ratio (SINR) requirement. Unfortunately, without constraints on the minimum received SINR requirements for FBSs' scheduled users (FUEs), the schemes in [6,7] lead to unnecessary transmissions and increase interference. This is because FBSs keep transmitting even though the received SINRs at FUEs are lower than their minimum received SINR requirements. In [8,9], the authors propose distributed power control algorithms that aim to maximise system capacity subject to satisfy minimum received SINR requirements for both MBS and FBSs. Here, the power control optimisation for FBSs are separated into two optimisation processes. In the first process, FBSs' transmit powers are optimised as if the MUEs do not exist. In the second one, a different algorithm is then used to reduce FBSs transmit power until both FUEs and MUEs can be satisfied. Although the first process has been shown to be optimal, there is no guarantee that power control solution that combine both processes are also optimal. As it will be shown later in the paper, this indeed lead to a non-optimal power solution. In addition, none of the existing schemes [6-9] consider cooperation among base stations (BSs) which has been shown to improve network performance significantly [11].

In this paper, we propose a new distributed downlink power control scheme for HetNets. We formulate the two processes used in [8,9] as a single optimisation function. This is done by formulating the global optimisation function that maximises the number of FBSs transmissions in a single HetNets wireless channel with FBSs transmit power as its variables and minimum received SINR requirements for FUEs and MUEs as its constraints. We then decompose this maximization problem into an individual FBS power control problem, solvable at a FBS level. We assume FBSs and MBS can communicate with a femtocell gateway (FGW) through internet backhaul connections [2]. A power control scheme for the FBSs is then proposed based on the stochastic learning

automaton technique [12]. Specifically, each FBS is treated as a learning automaton and maintains a probability vector to select its transmit power.

During the learning process, each FBS adjusts its probability vector based on the interference information, referred to as reward, from the FGW. The reward indicates the number of FBSs' transmissions that can be supported under the SINR requirement constraints of FUEs and MUEs. Simulation results show the proposed algorithm can achieve more than twice the number of simultaneous FBSs' transmissions achieved by existing APLS [7] and MDPC [9] algorithms. Our first contribution in this paper is that an FBS only transmits if its transmit power can satisfy the minimum received SINR requirement of its user, while guaranteeing the minimum received SINR requirements of MUEs. This eliminates unnecessary transmissions in [6-9] where FBSs are still transmitting even though the received SINRs of FUEs are very low. Secondly, unlike the schemes in [6-9] where each FBS optimizes its transmit power individually, the proposed scheme exploits FBSs cooperation where the SINR satisfaction information about the FBSs' transmissions are propagated to all FBSs through FGW. This results in a larger number of FBSs' transmissions that can satisfy SINR requirements of FUEs and MUEs as compared to the schemes in [6-9]. Thirdly, unlike schemes in [6-9] where transmit powers in HetNets are assumed to be continuous, we consider a discrete transmit power set which is commonly used in real digital cellular systems [13].

The remainder of this paper is organized as follows: Section II presents the system model. The optimal power control problem is formulated in Section III. Section IV describes the proposed stochastic learning based discrete power control algorithm. Section V discusses the simulation results. Finally, Section VI concludes this paper.

2. Methods

We consider a heterogenous network as shown in Figure 1. MBS 0 is located at the centre of a macrocell serving an area C with a radius of R_m . N FBSs denoted by $i \in N, N = \{1, 2, \dots, N\}$ are deployed within C . FBSs operate in CA mode and use the same spectrum band

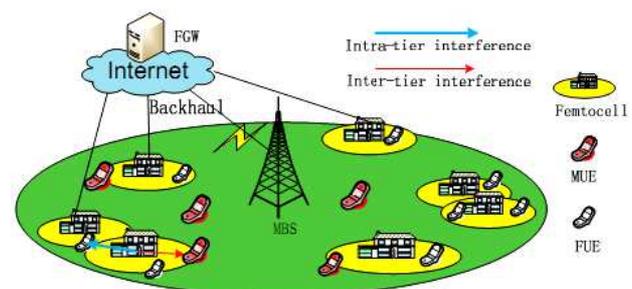


Figure 1. System Model for a Heterogeneous Network

with MBS 0. MBS and FBSs can communicate via a FGW through Internet-based IP backhaul [2]. Both MUEs and FUEs are randomly located within their serving base stations' coverage areas. We further assume that each base station serves one user at a transmission slot (a transmission slot may refer to a time resource over the spectrum band).

We assume that during a transmission slot, MBS 0 determines its transmit power q_0 based on its own power control policy. There are L different transmit powers for FBSs with the maximum transmit power q_{\max} and the minimum transmit power 0, defined as a set $Q = \{0, q_{\max}L-1, 2 \times q_{\max}L-1, \dots, q_{\max}\}$. Thus each FBS $i, i \in N$ chooses its transmit power q_i from Q . By letting MUE 0 and FUE $i, i \in N$, the received SINR of MUE 0 can be expressed as

$$\gamma_0 = \frac{g_{0,0}q_0}{\sigma^2 + \sum_{i \in N} g_{0,i}q_i} \quad (1)$$

and received SINR of FUE $i, i \in \{1, \dots, N\}$ as

$$\gamma_i = \frac{g_{i,i}q_i}{\sigma^2 + g_{i,0}q_0 + \sum_{j \in N, j \neq i} g_{i,j}q_j} \quad (2)$$

where σ^2 is the background noise power that is assumed to be the same for all users, and $g_{i,j}, 0 \leq i, j \leq N$ is the channel gain from BS j to UE i . We further define Γ_i as the minimum received SINR requirement of UE $i, i \in \{0, 1, \dots, N\}$ and define the following indicator function for each UE,

$$I_i = \begin{cases} 1, & \gamma_i \geq \Gamma_i \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where I_i indicates whether the received SINR at UE i satisfies its minimum SINR requirement or not. By using (3), the number of FBSs' transmissions that achieve the minimum SINR requirements at FUEs in the presence of MBS's transmission, N_s can then be written as,

$$N_s = I_0 \sum_{i=1}^N I_i \quad (4)$$

Note that $N_s > 0$ only when the received SINR of MUE 0 and at least one scheduled FUE are greater than or equal to their minimum received SINR requirements. By using (4) and letting $q = [q_1, q_2, \dots, q_N]$ be the selected transmit power vector of all FBSs, we can then

write the optimisation function for the power control problem as follows,

$$\max_q N_s \quad (5a)$$

$$s.t. \quad q_i \in Q \quad \forall i \in N \quad (5b)$$

where constraint (5b) indicates the choices of transmit power for FBSs, which defined by $Q = \{0, q_{\max}L-1, 2 \times q_{\max}L-1, \dots, q_{\max}\}$. Note that the global optimal solution for (5) can be obtained by using an exhaustive search over all the possible transmit power combinations for all FBSs and selecting the transmit power combination that gives the maximum of (5). Unfortunately, the exhaustive search scheme requires high computational complexity.

Stochastic learning based discrete power control.

In this section, we propose a stochastic learning technique based solution for (5) where each FBS is regarded as a learning automata that adjusts its own transmit power by using the reward containing interference information obtained from FGW. We will describe the basic of stochastic learning approach and how to use it to develop the proposed discrete power control algorithm.

Preliminaries of stochastic learning.

Stochastic learning automata are adaptive decision making devices that are capable of learning the desirable actions through interactions with the environment [12]. A stochastic learning automaton (SLA) can be represented by a tuple $\{A, p, u, T\}$ where $A = \{a_1, a_2, \dots, a_m\}$ is a finite action set of all possible actions a SLA can take. $p(k) = [p_1(k), p_2(k), \dots, p_m(k)]$, $k = 1, 2, \dots$ is the action probability vector at step k where $p_i(k)$ represents the probability for SLA to choose action $a_i \in A$ and $\sum_{i=1}^m p_i = 1$. $u(k)$ is the reward a SLA will get by taking action a_i , which should satisfy $0 < u(k) < 1$. Higher reward represents better action choice for a SLA. T is the learning scheme used by a SLA to update its action probability vector, based on its current action probability vector, its action, and the reward received. Figure 2 shows the interactions between a SLA and the environment. Specifically, the interaction at each step k consists of the following sequence.

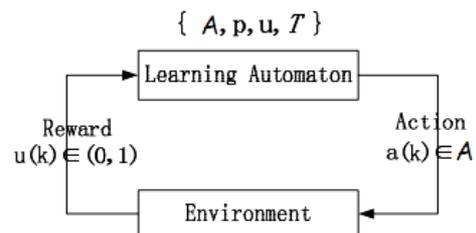


Figure 2. Learning Automaton

In general, a learning scheme is a function which can be represented as

$$p(k+1) = T(p(k), a(k), u(k))$$

- The SLA selects an action $a(k) \in A$ based on $p(k)$, here $\text{Prob}[a(k) = a_i] = p_i(k)$.
- Environment gives a reward $u(k)$ as a response to the action.
- Based on its action $a(k)$ and the reward $u(k)$ received, SLA updates its action probability vector $p(k)$ into $p(k+1)$ under the learning scheme T .

The process is repeated until p converges. The objective of a learning automaton is to find the optimal action which incurs the highest reward after several iterations. There are several learning schemes proposed in the literature, with which the learning automata can asymptotically learn the optimal action, such as linear reward-penalty (LR-P) schemes, linear reward-inaction (LR-I) schemes and some other non-linear schemes [17,18] (see [12] for a survey).

Stochastic learning based discrete power control algorithm. In our system, we regard each FBS as stochastic learning automaton. In the following, for each FBS i , $\forall i \in N$, we define its action set, probability vector, learning scheme and reward function. 1) Action set: An action for a FBS is a transmission using a selected transmit power. Here we use the selected transmit power $q_i \in Q$ to represent an action a_i of FBS i . There are L available transmit powers in power set Q . 2) Probability vector: Corresponding to the action set, the probability vector at step k is defined as: $p_i(k) = (p_{i1}(k), p_{i2}(k), \dots, p_{iL}(k))$, where $p_{ij}(k)$ represents the probability for FBS i to choose the j th transmit power at step k . 3) Reward: Based on our optimization objective defined in (5), the reward function for FBS i at step k is defined as,

$$u_i(k) = N_s / N \quad (6)$$

Here, N_s is the number of FBSs' transmissions that can be supported under the SINR requirement constraints of FUEs and MUEs, as defined in (4). In each step, based on the received transmit powers from all base stations, MUE 0 and FUE i , $i \in N$ will calculate and send their I_i (3) to FGW via MBS 0 and FBS i respectively. Then the FGW calculates N_s and normalizes it by N and broadcasts $u_i(k)$ to all FBSs. The objective of a learning automaton is to find the optimal action which incurs the highest reward. When FBSs can access the same wireless channel with MBS, where $\gamma_{i0} = 1$, reward function is a monotonically increasing function of the number of FBSs' transmissions with their minimum SINR requirements satisfied. This will encourage FBSs

to adjust their transmit powers in order to achieve the biggest N_s which incurs the highest reward. The maximum reward can be obtained when the SINR targets of both MUE 0 and all the scheduled FUEs can be met, where $N_s = N$. 4) Learning scheme: Learning scheme is an algorithm used by FBSs to update their probability vectors $p_i(k)$. Here, we choose the following scheme:

$$p_i(k+1) = p_i(k) + \theta u_i(k) (e_i - p_i(k)) \quad (7)$$

where $0 < \theta < 1$ is the learning rate, e_i is a unit vector of L dimension with i th component be unity, which indicates FBS i chooses the i th transmit power from Q . This learning scheme is known as linear reward-inaction LR-I scheme [12]. LR-I is a well-known updating scheme in the stochastic learning theory. It has been proven to be ϵ -optimal with a strong convergence property in [12,22], respectively. The proposed stochastic learning based discrete power control algorithm is described in Algorithm 1.

Algorithm 1: Stochastic Learning based Discrete Power Control Algorithm (SL-DPC)

1: Set the initial transmit power selection probability vector $p_i(0)$ for each FBS i . Let $p_{i1}(0) = 1/L$, $\forall i \in N$, $1 \leq i \leq L$; 2: At every step k , each FBS i first chooses its transmit power $q_i(k)$ according to its transmit power selection probability vector $p_i(k)$, then transmits data to FUE i with the selected power $q_i(k)$; 3: Both MBS and FBSs send their indicators I_i , $0 \leq i \leq N$ (3) to the FGW, then each FBS i obtains reward $u_i(k)$ specified by (6) from the FGW; 4: Each FBS updates its transmit power selection probability vector according to the scheme specified in (7); 5: If $\forall i \in N$, there exists a transmit power selection probability $p_{il}(k)$, $1 \leq l \leq L$ which is approaching one, e.g., larger than 0.99 [22], then stop; Otherwise, go to step 2);

Convergence property of SL-DPC. We now discuss the convergence characteristic of the proposed power control scheme. We use the fact that a learning automata system with common payoff and LR-I scheme as in (7) always converges to a pure strategy Nash equilibrium (Refer to Theorem 3.1 and 4.1 in [22] for proof). In our system, based on (6) and (7), at step k , FBSs will receive the same reward value from FGW, i.e., $u_i(k) = u_j(k)$, $\forall i, j \in N$ and employ LR-I scheme. FBSs act as a learning automata system with a common payoff and LR-I scheme. Thus, the proposed scheme will always converge to a pure strategy Nash equilibrium which is also a local optimal solution for (5).

3. Results and Discussion

In this section, we evaluate the system performance of the proposed SL-DPC algorithm and compares its system performance with the global optimal exhaustive

search scheme, the APLS scheme [7] and MDPC scheme [9], where FBSs control their transmit powers without any cooperation and without guaranteeing the minimum received SINR requirements for FUEs.

Simulation setup. In this paper, we consider a HetNet deployed as shown in Figure 3. One macrocell with three sectors is considered. A femtocell block is deployed in each sector. The dual stripe model [14] is adopted to represent the dense-urban multi-femtocell deployment environment. As illustrated in Figure 4, a femtocell block is of size 120 m × 70 m and includes two buildings (we assume each building has only one floor). In each building, there are 2 × 10 apartments, which are of size 10 m × 10 m. Thus, there are 40 apartments in a block. A 10m width street lies between two buildings.

We assume the distance between Femtocell blocks in different sectors is sufficiently large so that the intra-tier interference between blocks can be neglected. In the simulation we consider a femtocell block whose block centre is at a distance of 200 m from MBS 0. $N(N \leq 40)$ FBSs are deployed in this block where each FBS is installed at the centre of an apartment. FUEs are randomly generated in apartments where a FBS is installed, while MUEs are randomly dropped in the block. Each base station transmits to a user in its coverage based on its own scheduling policy (e.g., proportional fair scheme). We assume the minimum received SINR requirement for FUEs and MUE are 20dB and 12.5dB respectively. We adopt the approach in [23] for modelling path loss in a dense-urban multi-femtocell environment. The step size for learning scheme is set to 0.1 according to [22], which can achieve good balance between the system performance and learning rate. The system parameters are summarized in Table 1 [8,9].

Convergence of the SL-DPC algorithm. We first show the convergence property of the proposed SL-DPC

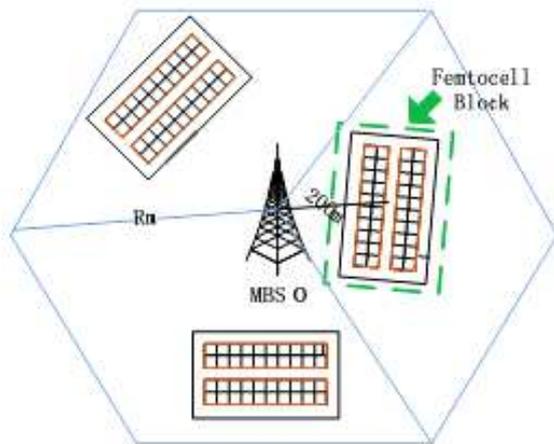


Figure 3. Simulation System Model with Dual Strip Model

algorithm. Here we consider there are $N = 10$ FBSs and $L = 4$ different transmit powers for each FBS, where

$$Q = \left\{ 0, \frac{q_{max}}{3}, \frac{2 \times q_{max}}{3}, q_{max} \right\}. \text{ We assume a static channel and user location scenario here.}$$

Table 1. Simulation Parameters

Parameters	Value
Macrocell radius (Rm)	1000m
System bandwidth	10 MHz
Carrier frequency (f)	2.0 GHz
Transmit power of macrocell (q0)	43 dBm
Maximum transmit power of femtocell (qmax)	13 dBm
Inner wall penetration loss factor (Liw)	10 dB
Outer wall penetration loss factor (Low)	refer to [23]
Path loss model	12.5 dB
Minimum received SINR for MUE 0 (Γ_0)	20 dB
Minimum received SINR for FUE i (Γ_i)	-174 dBm/Hz
White noise power density	(0, 1)
Learning step size (θ)	

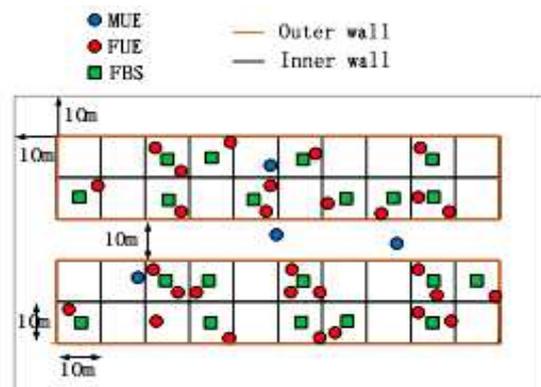


Figure 4. Dual Strip Model for Femtocell Block

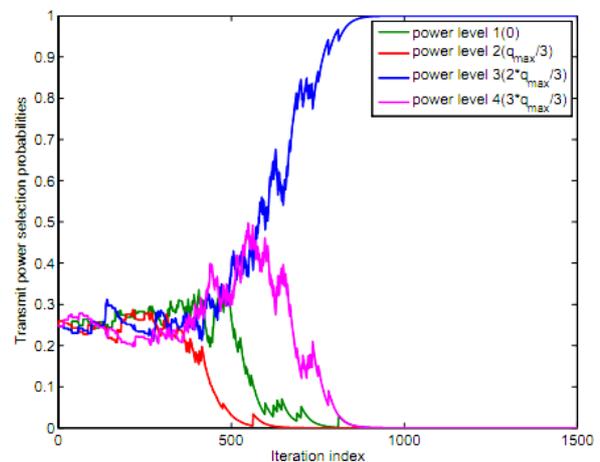


Figure 5. Evolution of the Transmit Power Selection Probability of an Arbitrary FBS. $N = 10, L = 4, \theta = 0.1$

Figure 5 shows the evolution of the transmit power selection probability of one of the FBSs, using the proposed algorithm under given simulation scenario. We can see that the transmit power selection probability vector for the FBS evolves from the initial values $\{1/4, 1/4, 1/4, 1/4\}$ to $\{0, 0, 1, 0\}$ in about 800 iterations.

Thus, this FBS finally selects power $\frac{2 \times q_{\max}}{3}$ with probability one. Similar observations are observed for other FBSs. This indicates that the transmit powers of FBSs converge to fixed values which indicate a stable solution for (5).

Figure 6 shows the evolution of the reward value (6) for FBSs during the calculation process of the proposed algorithm. We can see that the common reward for FBSs also converges to a specific value of 0.5 which is also a NE point according to the analysis of the convergence property in Section IV.

Performance comparison. Figure 7 shows the average number of FBSs' transmissions with the minimum SINR constraints satisfied of the proposed SL-DPC, APLS and MDPC schemes as the number of FBSs increases over 1000 trials where we vary the location of FBSs and MBS 0 users for each trial. Figure 7 shows that the proposed SL-DPC algorithm outperforms the APLS and MDPC schemes. The performance gain of the proposed algorithm over APLS and MDPC schemes is higher as with the number of FBSs increases. When there are 30 FBSs that want to transmit at the same time in the area, the number of active FBSs of the proposed algorithm is more than twice the numbers of APLS and MDPC schemes. This is because using the proposed scheme FBSs take the number of FBSs' transmissions that can be supported into consideration when selecting their transmit powers. In order to achieve a higher number, some FBSs will choose to shut down. This reduces both the inter-tier and intra-tier interferences,

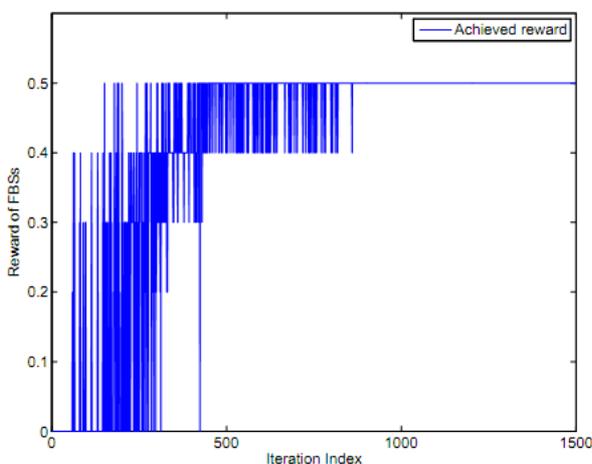


Figure 6. Evolution of the Reward for FBSs, $N = 10$, $L = 4$, $\theta = 0.1$

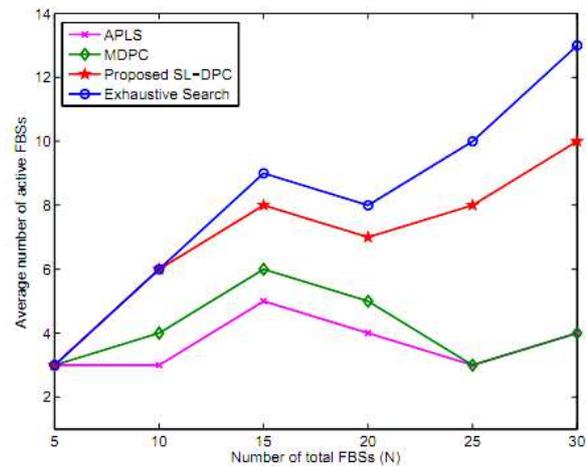


Figure 7. Average Number of Satisfied FBSs' Transmissions Comparison. $L = 4$, $\theta = 0.1$

thus more FBSs can meet their users' minimum SINR requirements. However, in MDPC and APLS schemes, FBSs maximize their own rates and adjust their transmit powers without guaranteeing the minimum SINR requirements of FUEs. That means those FBSs that can not satisfy minimum SINR requirements will still transmit to their users. That results in higher co-channel interference in the system and reduces the number of FBSs' transmissions whose SINR requirements can be satisfied. We also compare the number of simultaneous FBSs' transmissions that can be supported by SL-DPC algorithm with the exhaustive search scheme. As shown in Figure 7, the performance gap between exhaustive search and the SL-DPC algorithm becomes larger with the number of deployed FBSs increases.

This is because in the exhaustive search scheme, the central controller knows all the system parameters including channel states, number of FBSs and their available transmit powers. It finds the optimal solution by trying all the possible transmit power combinations. While using the SL-DPC scheme FBSs try possible transmit powers based on their probability vectors, which are evolved in a try-error-learn mode. There are L^N different transmit power combinations, with N increases, the number of combinations increases dramatically, it is harder for FBSs to try all the combinations, they are more likely to converge to a local optimal point instead of converging to the global optimal.

4. Conclusion

In this paper, we investigate a downlink discrete power allocation problem in HetNets with an objective of maximizing the number of active FBSs subject to the minimum received SINR requirements of the scheduled HetNet users. We propose a power allocation scheme based on stochastic learning automata technique where each FBS is treated as a learning automaton and

maintains a probability vector to select its action. The proposed scheme exploits the information about the number of FBSs that can satisfy the minimum received SINR requirements for their respective users to update the probability vector for selecting FBS transmit power. Simulation results showed that the proposed scheme can achieve a significantly higher number of active FBSs as compared to other schemes in the literature.

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