

Thermoeconomic Optimization of Cascade Refrigeration System Using Mixed Carbon Dioxide and Hydrocarbons at Low Temperature Circuit

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Abstract

Many applications and industrial processes require very low cooling temperature, such as cold storage in the biomedical field, requiring temperature below $-80\text{ }^{\circ}\text{C}$. However, single-cycle refrigeration systems can only achieve the effective cooling temperature of $-40\text{ }^{\circ}\text{C}$ and, also, the performance of the cycle will decrease drastically for cooling temperatures lower than $-35\text{ }^{\circ}\text{C}$. Currently, most of cascade refrigeration systems use refrigerants that have ozone depletion potential (ODP) and global warming potential (GWP), therefore, in this study, a cascade system is simulated using a mixture of environmentally friendly refrigerants, namely, carbon dioxide and a hydrocarbon (propane, ethane or ethylene) as the refrigerant of the low temperature circuit. A thermodynamic analysis is performed to determine the optimal composition of the mixture of carbon dioxide and hydrocarbons in the scope of certain operating parameters. In addition, an economic analysis was also performed to determine the annual cost to be incurred from the cascade refrigeration system. The multi-objective/thermoeconomic optimization points out optimal operating parameter values of the system, to addressing both exergy efficiency and its relation to the costs to be incurred.

Abstrak

Optimisasi Termoekonomi dari Sistem Pendinginan Cascade Menggunakan Campuran Karbon Dioksida dan Hidrokarbon di Sirkuit Suhu Rendah. Banyak aplikasi dan proses industri membutuhkan suhu pendingin yang sangat rendah, seperti *cold storage* di bidang biomedis yang memerlukan suhu di bawah $-80\text{ }^{\circ}\text{C}$. Namun, sistem pendingin siklus tunggal hanya dapat mencapai suhu pendinginan yang efektif $-40\text{ }^{\circ}\text{C}$ dan kinerja siklus akan menurun drastis untuk pendinginan dengan suhu lebih rendah dari $-35\text{ }^{\circ}\text{C}$. Saat ini, sebagian besar sistem pendingin cascade menggunakan refrigeran yang memiliki potensi penipisan ozon (ODP) dan potensi pemanasan global (GWP), oleh karena itu, dalam penelitian ini, sistem cascade disimulasikan menggunakan campuran refrigeran ramah lingkungan, yaitu karbon dioksida dan hidrokarbon (propana, etana atau etilena) sebagai refrigeran sirkuit suhu rendah. Sebuah analisis termodinamika dilakukan untuk menentukan komposisi optimal dari campuran karbon dioksida dan hidrokarbon dalam lingkup parameter operasi tertentu. Selain itu, analisis ekonomi juga dilakukan untuk menentukan biaya tahunan yang harus dikeluarkan dari sistem pendinginan cascade. Optimisasi multi-obyektif/termoekonomi menunjukkan nilai parameter operasi yang optimal dari sistem untuk mengatasi kedua efisiensi eksergi (*exergy*) dan kaitannya dengan biaya yang akan dikeluarkan.

Keywords: Thermoeconomic, cascade, carbon dioxide, hydrocarbons, multi-objective

1. Introduction

The single-cycle refrigeration system is not proper to be used for cooling temperatures lower than $-40\text{ }^{\circ}\text{C}$; therefore, cascade refrigeration systems are chosen for very low cooling temperature [1]. Cascade refrigeration system consists of two refrigeration systems or more that work independently. The refrigeration systems are connected with cascade-heat exchanger where the heat is released

by condenser at low temperature circuit (LTC) then the heat is absorbed by evaporator at high temperature circuit (HTC). Cascade refrigeration systems are also generally applied to areas such as pharmaceuticals, chemicals, blast freezing, thawing gas, aviation (aeronautics) and others [2]. To reach $-80\text{ }^{\circ}\text{C}$, low temperature circuit used CFC refrigerants such as R13 or R503, but in the year 2010, it has been banned for developing countries because the ozone depleting potential (ODP) [3].

HFC refrigerants, such as R23, are not responsible for ozone depletion effect; nevertheless, they have global warming potential (GWP). Therefore, alternative refrigerants, among natural refrigerants, with no or low ODP and GWP, are required; one of them is carbon dioxide [4]. Carbon dioxide has some advantages, namely not toxic, non-flammable, easily obtained, no ODP and very low GWP. However, the high pressure and the triple point temperature restrict the usage of carbon dioxide below $-56.6\text{ }^{\circ}\text{C}$. To overcome this short coming, carbon dioxide is mixed with other natural refrigerants, namely hydrocarbons. As refrigerants, hydrocarbons have good thermophysical properties. They are non-toxic and environmentally friendly. However, their flammability [5] needs to be carefully considered when the application case is selected and the system is designed. The mixing of hydrocarbons with carbon dioxide is expected to reduce the flammability of hydrocarbons, as well as the pressure and the triple point of carbon dioxide.

Among previous studies suggesting the use of carbon dioxide and hydrocarbon as refrigerants for cascade refrigeration systems, Bhattacharyya *et al.* (2005) used propane at low temperature circuit and carbon dioxide at high temperature circuit for further optimization of the cascade refrigeration system used for cooling and heating [6]. Alhamid M.I *et al.* (2010) also performed a thermodynamic simulation, followed by experimental investigation to determine the optimal composition of refrigerant mixture of carbon dioxide and ethane at low temperature circuit cascade refrigeration systems. The mixing gained a zeotropic composition (54% CO_2 and 46 % ethane) in mole fraction to achieve evaporation temperature of $-80\text{ }^{\circ}\text{C}$ [7]. However, the composition and scope of the operation parameters of several variations of the refrigerant mixture of carbon dioxide and hydrocarbons, which $\text{CO}_2/\text{R290}$, $\text{CO}_2/\text{R170}$, $\text{CO}_2/\text{R1150}$ at low temperature circuit cascade refrigeration system are not yet known.

Optimizing the design parameters and performance (measured in COP) of the combination CO_2 -ammonia performed to determine the optimum condensation temperature cascade condenser through thermodynamic analysis has been done by Lee *et al.* (2006) using a software developed by the International Institute of Refrigeration (IIR) [3]. Through the regression analysis, the optimal temperature of the CO_2 condenser and a maximum COP of CO_2 were obtained as a function of temperature evaporation T_E , ammonia condensation temperature T_C , and the temperature difference in cascade heat exchanger DT . Dopazo *et al.* (2009) [8] also performed thermodynamic analysis on a combination of ammonia- CO_2 similar to that carried out by Lee *et al.* (2006), but with a different entropic efficiency equation. The results of thermodynamic analysis derived two correlations to determine the optimal temperature in the

cascade condenser and maximum COP. Getu and Bansal (2008) [9] also conducted a similar study, but using Engineering Equation Solver software (EES) 2006 with the purpose of obtaining the design parameters and the optimal operation of the system by adding a super heating ΔT_{sup} and subcooling ΔT_{sub} . Thus, it was concluded that a multi linear regression analysis can be used to determine the optimum condensation temperature cascade condenser $T_{CAS; E; OPT}$, the maximum COP, and the ratio of the mass flow rate from R717 to R744 cascade refrigeration system.

Bingming *et al.* (2009) studied the combination of ammonia- CO_2 by experiment [10]. Further, the analysis of the thermodynamic equation COP_{max} by Lee *et al.* (2006) is considered more appropriate than the proposed by Dopazo *et al.* (2009). Dopazo *et al.* (2010) also conducted experiments to evaluate the cascade refrigeration system with CO_2 -ammonia for the freezing application. Studies related to the cascade refrigeration system with CO_2 -ammonia thermoeconomic on optimization and exergy analysis has been developed by Rezayan and Behbahaninia (2011) [11]. The results of the optimization showed the cooling capacity of 40 kilo watt constant would reduce the annual cost by 9.34%.

In this study, a thermodynamic and economic simulation of a cascade refrigeration system working with a mixture of CO_2 with three hydrocarbon refrigerants (propane, ethane and ethylene) at the low temperature circuit were performed. The optimum composition of the mixture of CO_2 and hydrocarbon refrigerants with evaporating temperature of $-80\text{ }^{\circ}\text{C}$ was defined. Further, thermodynamic, economic, and multi-objective optimizations for the operating parameters of the cascade refrigeration system at the optimum mass fraction composition were also compared.

2. Methods

Thermodynamic analysis. Thermodynamic calculations of cascade refrigeration system were done by simulating the thermodynamic states of the refrigerant. Theoretical thermophysical properties and composition of the refrigerant were obtained from REFRPROP version 8. The calculation and optimization were completed by MATLAB programming language. The composition of refrigerant mixture for low temperature circuit was directed at maximizing COP, provided that the carbon dioxide did not undergo crystallization and values capable of burning (flammability) of hydrocarbons were reduced.

The thermodynamic analysis conducted in this study relied on the following main assumptions: a). Compression process in the compressor was not isentropic, but expressed as a function of pressure ratio, b). Combined electrical and mechanical efficiency of each compressor was assumed to be 0.93 [4], c). Heat and pressure losses

in pipes and refrigeration system components were ignored, d). Expansion process in the expansion valve was assumed to be isenthalpic, e). Kinetic and potential energy were ignored, f). Dead state condition (when the system is in thermodynamic equilibrium with the environment) was assumed at 25 °C temperature and pressure of 101.3 kPa, g). The difference between the cold room temperature T_{cl} and temperature evaporation T_E was assumed to be 5 °C, h). The cooling capacity was assumed, namely 0.5 kW, i). The direction of heat flow into the system and work done by the system was positive, and vice versa.

Further, the equilibrium equation used to calculate the mass flow rate in the cascade cycle (Figure 1 and Figure 2), the compressor work, the rate of heat transfer from the condenser and heat exchanger cascade, the rate of entropy, and exergy loss rate are as follows:

Mass balance

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \quad (1)$$

Energy balance

$$\dot{Q} - \dot{W} = \sum \dot{m}_{out} \cdot h - \sum \dot{m}_{in} \cdot h \quad (2)$$

Exergy balance

$$\dot{X}_{des} = \sum_{out} \left(1 - \frac{T_0}{T_j} \right) \cdot \dot{Q}_j - \dot{W} + \sum_{in} \dot{m} \cdot \psi - \sum_{out} \dot{m} \cdot \psi \quad (3)$$

The cascade refrigeration system was optimized by determining the values of operating parameters that maximize the exergy efficiency achieved and the minimizing cost incurred. In this way, the thermodynamic approach follows criteria based on efficiency and economy. The determination of the optimum operating conditions of the cascade refrigeration system is determined by the operating parameters that most influence the state of the system, called the decision variables.

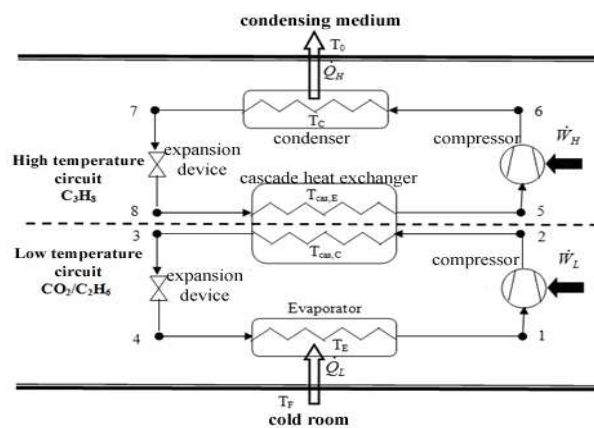


Figure 1. Schematic Cascade Refrigeration System

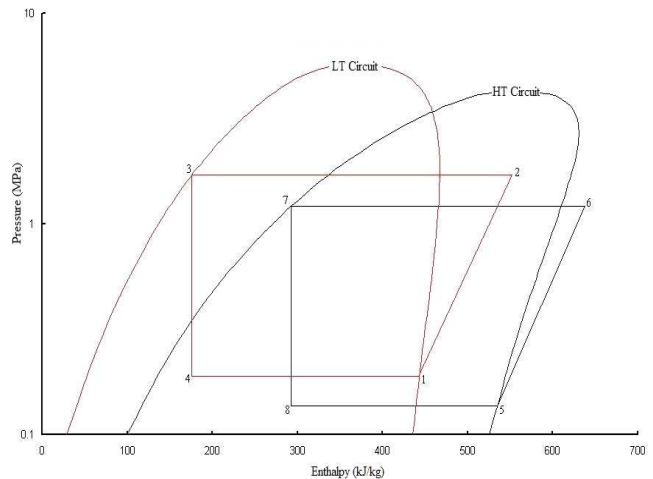


Figure 2. Points on the State of the System Cascade Diagram p-h

In accordance with its name, the value of the decision variables determines the state of the system as a whole, both energy and economic aspects. In this study, the operating parameters used as decision variables of the cascade refrigeration system consisted of temperature evaporation T_{EVAP} , the condensation temperature of the low temperature circuit $T_{CAS,C}$, the temperature of the condenser T_{COND} , and the temperature difference in cascade heat exchanger DT .

The optimization method was driven towards the solution by varying the four decision variables in the scope of a certain value, then towards a combination of the four decision variable values that could produce the optimum conditions. The optimization scheme was done by optimizing first the exergetic efficiency of the system, followed by the cost of the system; in this case, the optimization was done by single-objective. The optimum results of the single-objective optimization will be used as four decision variables of the objective function. However, the expected results of the optimization procedure should not only correspond to a system that is thermodynamically optimized, but also optimum for economic factors. Therefore, multi-objective optimization was performed to obtain thermoeconomic optimal operating conditions, at which the cascade refrigeration system could work with high exergy efficiency, as well as low operative cost.

Constraints that existed on the optimization of cascade refrigeration system were the cooling capacity (cooling load) of 500 Watt, the environmental temperature $T_{ambient}$ at 25 °C, and the temperature difference. Evaporative cooling temperature T_{drop} was allowed at 5 °C. Coverage of the parameter values into the decision variables varied according to the area of operations allowed on a cascade refrigeration system, based on the

operating conditions common in cascade refrigeration system: the evaporation temperature T_{EVAP} was between 80 °C to 90 °C, the temperature of the condensation temperature circuit low $T_{CAS,C}$ was between -40 °C to 0 °C, the temperature of the condenser T_{COND} was between 30 °C to 40 °C, and the temperature difference in cascade heat exchanger was between 1 °C to 15 °C.

Process optimization to find the optimal parameter values was performed using Matlab program assistance, namely multi-objective (multi-objective optimization using Genetic Algorithm). The optimization method was done by the iterative process that follows the principles of evolutionary biology. The result of a multi-objective optimization is not a single solution, but rather a set of solutions or the optimum values of the decision variables that form a population of solutions. Any solution of multi-objective optimization result is an optima value, and not a single optimal solution that is more than other solutions. Therefore, in this study, the selection of the solution was determined by trade-offs of the decision makers, whether thermodynamic aspects were more emphasized or economic aspects should be considered.

Thermoeconomic analysis. The influence of the system parameters on energy prices paid was estimated by considering the economic analysis of each component in the cascade refrigeration system. Economic factors in this study were limited to two terms, namely capital cost and operational cost. In this study, equations and variables used in determining the capital and operating costs of a component were flexible, meaning that they could change any time depending on the dynamics of economic conditions and other factors.

In this study, the economic analysis conducted on the cascade refrigeration system suggested was carried out using the following assumptions: a). Life time of the equipment, n was assumed to be 10 years, b). The assumed total operating time of the system (h) was 7000 hours per year, c). The cost of electricity (C_{el}) was assumed to be \$ 0.12 per kWh, d). The rate of interest rates (i) was assumed to be 8%, e). Expansion device purchase costs were ignored because they were small compared to the overall cost of the system.

The equation for calculating the estimated costs arising from this system can be written as the cost of each component of the system, as follows: [13]

The cost of the compress was expressed as a function of the work input (in kW). For high temperature circuit, $C_{Comp,H}$:

$$C_{comp,H} = 9624,2 W_{comp,H}^{0,46} \quad (4)$$

whereas the purchase price of the compressor to the low temperature circuit, $C_{Comp,L}$:

$$C_{comp,L} = 10167,5 W_{comp,L}^{0,46} \quad (5)$$

Heat exchanger price was expressed as a function of capacity (kW) and broad dimensions of heat exchanger (A_o , in m^2).

For high-temperature circuit, price of condenser:

$$C_C = 1397 A_{0,C}^{0,89} + 629,05 W_{F,C}^{0,76} \quad (6)$$

For the low temperature circuit, evaporator price:

$$C_E = 1397 A_{0,E}^{0,89} + 629,05 W_{F,E}^{0,76} \quad (7)$$

The condenser and the evaporator fan were assumed as 50 W each. Heat exchanger price:

$$C_{CQS,C} = 2382,9 A_{O,CQS,C}^{0,68} \quad (8)$$

The area of each heat exchanger can be written as a function of the rate of heat transfer Q , heat transfer coefficient U_o , and the temperature difference between the heat exchanger with its environment ΔT as follows: [11]

$$A_o = \frac{Q}{U_o \Delta T} \quad (9)$$

Heat transfer coefficient of each heat exchanger was fixed as a constant value [11] 18.03 $W/m^2 \cdot K$ for evaporator was 6.85 $W/m^2 \cdot K$ for the condenser was 64.87 $W/m^2 \cdot K$ for cascade heat exchanger. The price of the above components was in units of dollars (U.S. \$). From the above equations, the total annual cost can be calculated as follows:

$$C_{total} = \left[C_{comp,H} + C_{comp,L} + C_C + C_E + C_{cas,C} \right] CRF + C_{ei,H} \left[W_{comp,H} + W_{comp,L} + W_{F,C} + W_{F,E} \right] \quad (10)$$

The variable CRF (Capital Recovery Factor) converts the present value of the initial investment into a stream of equal annual payments at fixed interest, and it can be defined by eq. (11).

$$CRF = \frac{i \times (1+i)^n}{(1+i)^n - 1} \quad (11)$$

where i is an interest rate, n is the life time of the system.

3. Results and Discussion

The optimization procedure was conducted after the optimum composition of each mixture of carbon dioxide and hydro carbons were obtained. Figure 3, Figure 4, and Figure5 show the optimum compositions of refrigerant mixture of CO₂ with propane, ethane, and ethylene respectively.

From Figure 3, it is possible to observe that the optimum composition of carbon dioxide and propane is obtained for a mass fraction of 94% carbon dioxide and 6% propane. Then, the optimum mixture of carbon dioxide and ethane corresponds to 64% carbon dioxide and 36% ethane. The mixture of carbon dioxide and ethylene has an optimal mixture composition at 37% carbon dioxide and 63% ethylene.

Thermoeconomic optimization involves simultaneously multi-objective thermodynamic functions and economic functions. Thus, the result of this optimization is a trade-off between the exergy efficiency and expenditure.

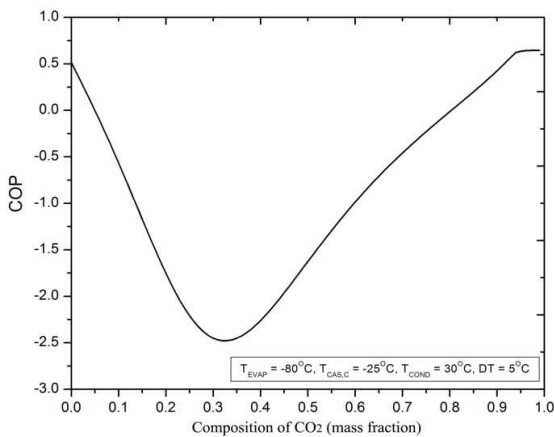


Figure 3. Effect of the Composition of the Mixture of CO₂ and Propane to COP

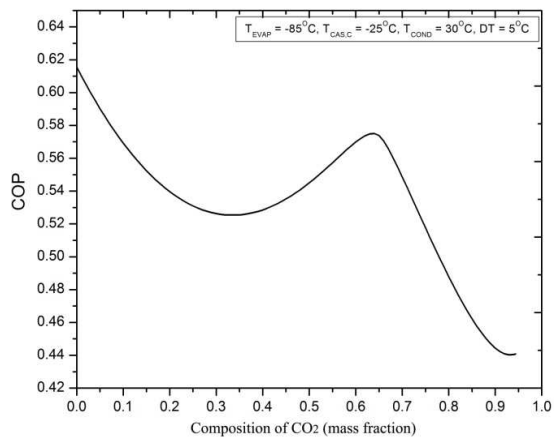


Figure 4. Effect of the Composition of the Mixture of CO₂ and Ethane to COP

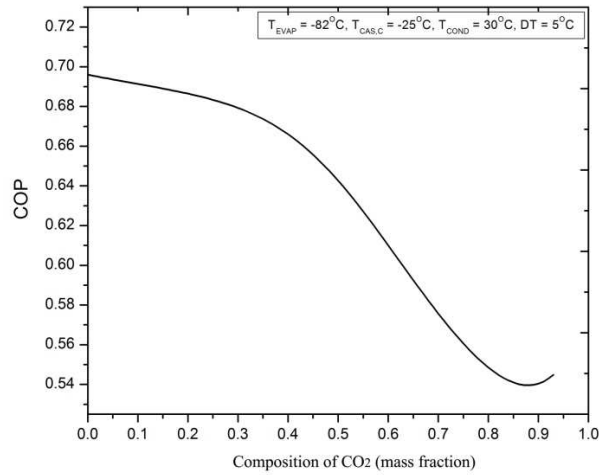


Figure 5. Effect of the Composition of the Mixture of CO₂ and Ethylene to COP

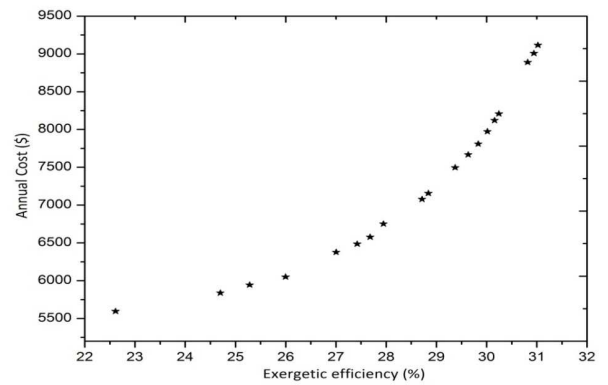


Figure 6. Pareto Frontier for CO₂ and Propane Mixture (94/6)

Figure 6, Figure 7, and Figure 8 show a set of solutions obtained from a multi-objective optimization that describes a relationship of mutual attraction between exergy efficiency (%) and annual costs (\$), referred to as optimum solutions or pareto frontier.

The optimum operating conditions for each type of refrigerant mixture of carbon dioxide and hydro carbons, determined at the optimum mass fraction composition, are as follows (Table 1-3).

By comparing each type of refrigerant mixture between carbon dioxide and hydro carbons, it was obtained that the cascade refrigeration system could be designed to achieve the evaporation temperature around -80 °C with propane as a refrigerant in high temperature circuit, whereas the proper refrigerant to be used in low temperature circuit was a mixture of carbon dioxide and ethylene with composition mass fraction of 37% carbon dioxide and 63% ethylene.

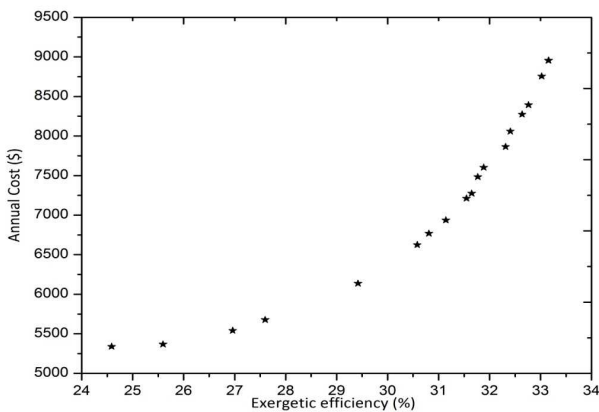


Figure 7. Pareto frontier for CO₂ and Ethane Mixture (64/36)

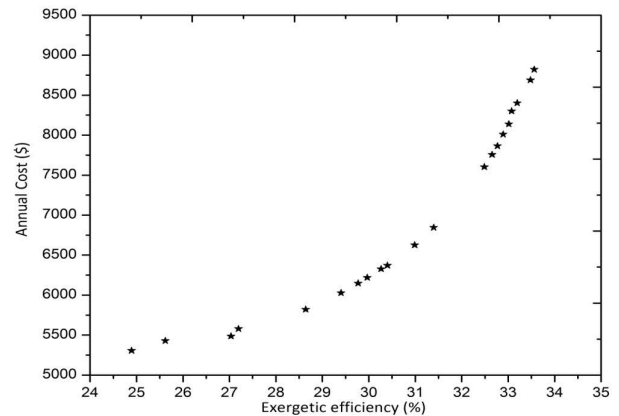


Figure 8. Pareto Frontier for CO₂ and Ethylene Mixture (37/63)

Table 1. Design of the Optimum Operating Conditions Cascade Refrigeration System Using Refrigerant Mixture CO₂/propane (94/6) at Low Temperature Circuit

Operating Parameter CO ₂ /Propane	Thermodynamic Optimized	Economic Optimized	Thermoeconomic Optimized
Temp. of evaporation, T_{EVAP} (°C)	-80	-80	-80.018
Temp. of cascade condenser, $T_{CAS,C}$ (°C)	-33	-30.5	-32.85
Temp. of condenser, T_{COND} (°C)	30	40	31.13
Temp. diff.cascade, DT (°C)	1	5.7	1.53

Table 2. Design of the Optimum Operating Conditions Cascade Refrigeration System Using Refrigerant Mixture CO₂/ethane (64/36) at Low Temperature Circuit

Operating Parameter CO ₂ /Ethane	Thermodynamic Optimized	Economic Optimized	Thermoeconomic Optimized
Temp. of evaporation, T_{EVAP} (°C)	-80	-80	-80
Temp. of cascade condenser, $T_{CAS,C}$ (°C)	-30.3	-27.8	-27.8
Temp. of condenser, T_{COND} (°C)	30	40	32.5
Temp. diff.cascade, DT (°C)	1	6.01	1.56

Table 3. Design of the Optimum Operating Conditions Cascade Refrigeration System Using Refrigerant Mixture CO₂/ethylene (37/63) at Low Temperature Circuit

Operating Parameter CO ₂ /Ethylene	Thermodynamic Optimized	Economic Optimized	Thermoeconomic Optimized
Temp. of evaporation, T_{EVAP} (°C)	-80	-80	-80.1
Temp. of cascade condenser, $T_{CAS,C}$ (°C)	-29.7	-26.95	-28.9
Temp. of condenser, T_{COND} (°C)	30	40	32.9
Temp. diff.cascade, DT (°C)	1	6.2	1.8

4. Conclusions

To overcome the operative limit of carbon dioxide, related to its triple point, CO₂ was mixed with hydro carbons. This widened the operative range of carbon dioxide by lowering pressure and triple point, together with a lower flammability than pure hydro carbons. Simulation results showed that each combination of CO₂

with propane, ethane, or ethylene had the possibility to work as a good refrigerant mixture, but from a thermoeconomic and first principle efficiency point of view, only the combination of CO₂ with ethylene had good prospect for the performance of the system. With respect to this criterion, the suggested combination value was 37% carbon dioxide and 63% ethylene. The thermoeconomic optimization characterized the relationship between the

system annual cost and its exergetic efficiency, and was used to select the four decisions value that could be used as set point to reach the best performance of the cascade refrigerant system using propane and CO₂/ethylene. In this condition, the calculated COP of the system reached a value of approximately 0.65.

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