Wavelet Transform on Digital Rainbow Hologram based on Spectral Compression for Quality Enhancement in 3D Display Media

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Abstract

A digital rainbow hologram (DRH) is a potential next-generation three-dimensional display media for the development of modern and smart electronics devices. It is one of the methods that can support the characteristic whereby a realistic display media occupies the space that the real object would have occupied. Since a rainbow hologram records a large amount of spatial or temporal frequency component from the object that represents the rainbow spectrum, a large amount of information needs to be decoded digitally. In this paper, to reconstruct a DRH, we propose a novel method based on the modulation of red, green, and blue spectral components of light by wavelet transform (WT) in the recording and reconstruction processes, which we digitally simulated in a computer using an algorithm. In the simulations, continuous WT (CWT) was based on Haar, Daubechies, Meyer, and Coiflet wavelets with a level set to be two. Based on the results of simulations using CWT, the optimum distance between object and hologram was 30 cm, and the maximum compression was 88.55%, which was achieved with Meyer wavelet. Moreover, optimal de-noising and optimal localization of spatial frequency component based on red, green, and blue spectral components were also achieved using the proposed method.

1. Introduction

Digital rainbow hologram (DRH) is a potential method to develop a 3D media display, and it can be implemented in the next-generation 2D or 3D schemes. It has a wide area of application as regards electronics devices [1-3]. Rainbow holograms have been implemented in many areas, such as image analysis in engineering, medical,
computing, entertainment, and mobile multimedia development. A hologram media has specific characteristics absent in others [4]. The advantage of DRH is that a wide spectrum of signal from the object can be recorded in the order of wavelength in spatial–temporal domain digitally and in real time. This means DRH offers a complete recording technique for acquiring the signal from the object rather than conventional hologram. Since it offers a complete recording technique, it requires decoding and compressing data when a huge amount of information is involved.

Regarding the advantage of wavelet transform (WT) as a tool for signal processing and analysis, digital holograms have been investigated, and so far algorithms that implement WT have been developed. In [5], one-dimensional and 2D Gabor wavelet transform (GWT) were implemented for image reconstruction in a digital hologram, where the result was the elimination of the effect of the zero-order term and the twin-image without spatial filtering. In [6], Fresnellet and GWT were implemented to reconstruct a digital hologram image, where the significant result was good localization properties of GWT bases in the space–frequency domain for view-based compression techniques for digital hologram reconstruction. In [7], a WT method was successfully implemented for digital hologram image reconstruction to achieve high image compression through the removal of the speckle noise, and hence, the signal to noise ratio (SNR) was significantly improved. In [8], a one-level WT-based method was implemented to achieve high image compression in a digital hologram. In [9], a recorded hologram from a charge-coupled device was improved by digital reconstruction via WT to decrease the speckle noise existing in the image.

Digital rainbow hologram is an imaging technique that simulates recording and reconstruction process in real-time holography by using the white light spectrum. White light spectrum can be decomposed as red (R), green (G), and blue (B) spectra since it is possible to combinatorially implement a coherent light. To simulate an optical process of rainbow hologram recording, many methods have been developed following advancements in digital computing. Such methods include the look-up table technique and filter banks that implement fast Fourier transform (FFT) as the major method for digitally compressing information [10,11]. However, these methods have a major disadvantage: an image’s resolution degrades significantly since there is no technique for compressing the RGB spectral components. Generally, the aforementioned methods spatially decode the information of the recorded object without considering the spectral component that is transmitted by the white light spectrum from the object. Considering the storage of the data of an image reconstructed from a digital hologram in a computer, the previous methods are limited when a large amount of information is involved.

Those methods have a major issue as regards the quality of the reconstructed image, as speckle noise exists and SNR also degrades.

Many methods have also been developed to compress a large amount of information from a rainbow spectrum. To achieve compression in DRH, the main focus should be minimizing the loss of the hologram characteristic as a 3D display media. Some of compression methods for rainbow spectrum are fringes decoding, temporal and spatial frequency decoding, and stationary decoding techniques. However, in the recording process, these compression techniques do not consider information from the object to be decomposed based on the major components of white light, which are R, G, and B; as a result, the information of the object to be recorded is limited, and information is lost in the reconstructed image. In other words, the losing of information for recording means the losing of resolutions in the reconstruction process. This problem causes image quality degradation, especially for the depth resolution of the reconstructed image. Unfortunately, it also reduces the characteristic of wide field-of-view resolution. For example, for compression by limiting the field of view from an object for video holography, where the object moves in the time, the resolution for spatial–temporal frequency is simplified through the aforementioned technique.

Regarding a DRH with a large amount of information, an efficient and effective method should be developed by considering the real characteristics of real-time holography with a 3D resolution. The method used to compute information in a DRH should include a specific technique that can implement compression without sacrificing the rainbow spectrum of an object so that the quality of the reconstructed image does not degrade. Wavelet transform is considered in the method proposed in this study. Through the use of WT in many schemes such as Mexican hat, Daubechies, and Morlet, the localization of spatial–temporal frequency in objects is realized via spatial and spectral decomposition, where red, green, and blue components, which are based on a rainbow spectrum, are used. Wavelet transform that has properties of scaling and shifting technique is used to perform compression in the object and fringe pattern of rainbow holograms. Through this compression, the optimal DRH quality is achieved.

In this paper, we propose a novel technique for DRH compression based on R, G, and B spectra in the recording and reconstruction processes. We implement continuous WTs (CWT) method in the reconstruction process through simulation. We also analyze the characteristics of the reconstructed image through the amount of information and histograms. The main technique for implementing WT in DRH is based on the R, G, and B spectral modulation components of white
light spectrum. Since a large amount of frequency components exists in a white light spectrum, in the simulation, the authors considered using spectral components that were 650 nm, 550 nm, and 450 nm as the representation of R, G, and B spectra, respectively. These three spectral components of light were used mainly because of the availability of coherent light or laser on those spectra that are widely produced by vendors such as an optical company or market, since DRH is mainly based on the interference process of coherent light. The main aim of this preliminary research is to investigate and analyze the performance of CWT in compressing, de-noising, and localization to modulate R, G, and B spectra in a DRH image in the reconstruction process.

2. Methods

Digital Rainbow Hologram and Wavelet Transform. The recording process for rainbow hologram is shown in Figure 1. An object acts as a light source when a coherent light composed of red, green, and blue spectra falls upon it. The reflected rainbow spectrum from the object that goes to recording plane, which is a holographic plate or film, interferes with a rainbow light from a point source. The rainbow spectrum from a light source is considered as the reference light, which does not contain information about spatial–temporal frequency from the object. The information from an object that is represented as the spatial–spectral component is modulated by the reflected rainbow spectrum and goes to the recording plane. In the cross section between the reflected light and reference light, an interference occurs, where a huge amount of information in the order of wavelength-based rainbow spectral component is recorded as the fringe pattern in the holographic film or plate.

The fringe pattern, which represents the rainbow spectrum of this recording process, lies within the $x$ and $y$ coordinates of the holographic film or plane, as stated in [12,13].

$$
\psi_o(x, y)\psi_r^*(x, y) + \psi_r^*(x, y)\psi_r(x, y) = \exp \left( \frac{-jk_o x o}{z_o} + \frac{j k_o x o^2}{2 z_o} \right) \\
\times \mathcal{F} \left\{ \sigma_o(x, y) \exp \left[ \frac{-jk_o}{2 z_o} (x^2 + y^2) \right] \right\} \\
+ \exp \left( \frac{j k_o x o}{z_o} - \frac{j k_o x o^2}{2 z_o} \right) \\
\times \mathcal{F} \left\{ \sigma_r(x, y) \exp \left[ \frac{-jk_o}{2 z_o} (x^2 + y^2) \right] \right\}
$$

where $\psi_o(x, y)$ and $\psi_r^*(x, y)$ are the reflectance rainbow spectra of the object for the real and complex components, respectively; $\psi_r(x, y)$ and $\psi_r^*(x, y)$ are the reference rainbow spectra of a point source for the real and complex components, respectively; $\mathcal{F} \left\{ . \right\}$ is the Fourier transform; and $k = 2\pi/\lambda$ is the wavelength number. In Eq. (1), the information of spatial frequency from the object is $k_o = k_o x / z_o$ and $k_y = k_o y / z_o$.

Basically, the frequency components of $k_x$ and $k_y$ are composed of red, green, and blue spectra. A large amount of information of the respective spectra are recorded in the holographic plate as a hologram.

For image reconstruction, the hologram plate is illuminated with a laser source that is considered as the rainbow light point from a distance $z_o$ as the recording process, as shown in Figure 2.

The reconstructed rainbow hologram image has properties of virtual and real images. Thus, the information from the reconstructed image is in the form of a rainbow spectrum based on spatial and spectral components of the recorded object, as stated in [13,14].
\[ \mathcal{F}\{\psi_0(x, y)\psi_0^*(x, y) + \psi'_0(x, y)\psi'_0(x, y)\} \propto \sigma_o \left( \frac{z_o}{f} x + z_o \frac{z_o}{f} y \right) \times \exp \left\{ -j k_o \left[ \left( \frac{z_o}{2z_o} x - x_o \right)^2 + \left( \frac{z_o}{2z_o} y \right)^2 \right] \right\} \]

\[ + \sigma_o^2 \left( \frac{z_o}{f} x + x_o \frac{z_o}{f} y \right) \times \exp \left\{ j k_o \left[ \left( \frac{z_o}{2z_o} x - x_o \right)^2 + \left( \frac{z_o}{2z_o} y \right)^2 \right] \right\} \]  

\[ \times \exp \left\{ -j k_o \left[ \left( \frac{z_o}{2z_o} x - x_o \right)^2 + \left( \frac{z_o}{2z_o} y \right)^2 \right] \right\} \]  

(2)

Based on Eqs. (1) and (2), WT can be implemented to achieve compression in a hologram without degrading the quality of the reconstructed image. The CWT properties allow for continuous localization in spatial frequency signal and red, green, and blue spectral components (RGB spectra) from an object. The mother wavelet for CWT is given as follows [15].

\[ \Psi(x) = \frac{1}{\sqrt{|s|}} \int x(t) M^* \left( \frac{t - \tau}{s} \right) dt, \quad (3) \]

where \( M \) is the transform function for scale \( s \) and translation \( \tau \), and \( x(t) \) is the fringe pattern in the holographic plate and is considered to be the signal for a higher resolution in 2D. This localization function is the WT that operates to localize the fringe pattern through translation and scaling within the spatial frequency component. The compression through this localization which is mother wavelet of CWT should be modulate the fringe pattern two-dimensionally, where the translation and scaling functions obey the Shannon theory; that is, they are more than twice the sum of spatial frequency component and RGB spectra from the object.

Thus, regarding Eqs. (2) and (3), the CWT to decompose the reconstructed image in R, G, and B spectra are stated below:

\[ \Psi(R_x) = \frac{1}{\sqrt{|s|}} \int x_R(t) M^* \left( \frac{t - \tau}{s} \right) dt, \quad (4) \]

\[ \Psi(G_x) = \frac{1}{\sqrt{|s|}} \int x_G(t) M^* \left( \frac{t - \tau}{s} \right) dt, \quad (5) \]

\[ \Psi(B_x) = \frac{1}{\sqrt{|s|}} \int x_B(t) M^* \left( \frac{t - \tau}{s} \right) dt, \quad (6) \]

where \( x_R(t), x_G(t), \) and \( x_B(t) \) are the decompositions of signal spectra from the reconstructed image as stated in Eq. (2). Then the overall signal of DRH can be stated as follows:

\[ \Psi(R, G, B)_x = \Psi(R)_x + \Psi(G)_x + \Psi(B)_x. \quad (7) \]

In Eq. (7), the overall signal of DRH has many advantages for digital signal processing: the information based on spectral components can be selected regarding the requirement for signal analysis, image brightness can be enhanced, noise can be minimized, speckles can be removed, and large amounts of information can be compressed. Thus, WT implementation in DRH can improve the characteristic of digital hologram as a 3D display media.
Simulation. To simulate the DRH recording process, an object of bitmap image (Figure 3) with a resolution of 256 × 256 was used. The object is an image that contains two letters, U and D, as shown in Figure 2.

This image was simulated with coherent light illumination with laser at several wavelengths of 650 nm, 550 nm, and 450 nm as the R, G, and B spectral components, respectively, as shown in Figure 4. It was illuminated with coherent light at a distance $z_1$ from the holographic film, which was the recording plane. Meanwhile the distance from the reference light source to the object was $z_2$, and the distance from the hologram plate to the object was $z_3$. Thus, the reflected light from the object, which was composed of R, G, and B, interfered with the reference light from a point source as illustrated in Figure 1. The light from this reference was considered to be a planar wave. The result of the interference light of R, G, and B spectra in the recording process was the fringe pattern in 2D. The fringe pattern was the DRH that was stored in the computer as data or information.

Subsequently, CWT was implemented in the fringe pattern to modulate the spatial frequency component and R, G, and B spectra from an object by scaling and shifting the spatial frequency component in high resolution. Thus, the result of modulation by CWT was a fringe pattern of high resolution localized using a window function and stored in a computer. For reconstruction, the modulated fringe by CWT is illuminated with a coherent light as the recording process; meanwhile, the reconstructed images are virtual and real images. The reconstructed images of DRH are analyzed in compression ratio and histogram properties.

In the simulation, $z_2$ and $z_3$ were set to be constant, while $z_1$ was set to be variable. The distance $z_1$ was set to be variable to investigate the performance of depth resolution, which is a characteristic of 3D display media. The WTs used in the simulation were Haar, Daubechies, Coiflet, and Meyer wavelets.

![Figure 3. Image Used as an Object for DRH Recording Process](image)

3. Results and Discussion

For the simulation, the object used is a 2D image with resolution of 256 × 256 for 8 bit with an extension of BMP as shown in Figure 4. The image is in grayscale format and contains letters U and D. In the simulation, $z_1$ and $z_2$ are 30 cm and 60 cm, respectively. The simulation results for DRH image reconstruction with various $z_2$ of 10 cm, 20 cm, 30 cm, 40 cm, and 50 cm are shown in Figures 5–9.
Figure 5. Reconstruction of DRH Image at $z_3 = 10$ cm: (a) DRH as the Combination of light Illumination at RGB spectra; (b) Measurement of Histogram of DRH Image Reconstruction

Figure 6. Reconstruction of DRH Image at $z_3 = 20$ cm: (a) DRH as a Combination of Light Illumination at RGB Spectra; (b) Measurement of Histogram of DRH Image Reconstruction

Figure 7. Reconstruction of DRH Image at $z_3 = 30$ cm: (a) DRH as a Combination of Light Illumination at RGB Spectra; (b) Measurement of Histogram of DRH Image Reconstruction

Figure 8. Reconstruction of DRH Image at $z_3 = 40$ cm: (a) DRH as a Combination of Light Illumination at RGB Spectra; (b) Measurement of Histogram of DRH Image Reconstruction
Based on the results of DRH image reconstruction in Figures 5–9, by modulation of RGB spectral components, image could be displayed for $z_3$ distances of 10–50 cm. The histograms for these various $z_3$ also show that the information displayed by the DRH image reconstruction is dominated by black pixels, which means that letters “U” and “D” have a very minor distribution in comparison. The black pixels represent non-interference coherent light source, which means they do not contain the information of the object. Thus, the modulation by CWT yields a fringe that contains only the information of coherent light source interference based on the RGB spectral components from the object. Figures 5–9 also show that $z_3 = 30$ cm is the optimum distance from the object to the hologram plate (the recorder of fringe). Meanwhile, other $z_3$ values show degradation in modulation of RGB spectral component. This is confirmed by the blur DRH image reconstruction at $z_3 = 50$ cm.

In Figure 10, DRH image reconstruction for $z_3 = 30$ cm is analyzed by CWT to investigate the performance of compression. Based on Figure 10, in the case of DRH image reconstruction for $z_3 = 30$ cm, compressions by CWT based on Haar, Daubechies, Meyer, and Coiflet wavelets showed moderate performance. These CWTs were set in a level 2. The optimum performance, a compression ratio of 88.55%, was achieved using Meyer wavelet. Other CWTs achieved compression ratios approaching 100%, which means they were not efficient. The compression by Meyer wavelet resulted in a slightly blurred image. This blur occurred because during compression, spatial information was taken from the object, which means some pixels in the object were decreased. However, Meyer wavelet could still display the information content clearly, with a compression ratio of 88.55%.

Figure 9. Reconstruction of DRH Image at $z_3 = 50$ cm: (a) DRH as a Combination of Light Illumination at RGB Spectra; (b) Measurement of Histogram of DRH Image Reconstruction

Figure 10. Compression of DRH Image at $z_3 = 30$ cm with Level 2: (a) Haar Wavelet; (b) Daubechies Wavelet; (c) Meyer Wavelet; (d) Coiflet Wavelet
The DRH image reconstruction by CWT for $z_3 = 30$ cm was analyzed to investigate the performance of de-noising or removing the noise that rises in the reconstruction process, and the results are shown in Figure 11.

Based on Figure 11, de-noising could be successfully performed via CWT based on Haar, Daubechies, Meyer, and Coiflet wavelets with level 2 so that the noise was removed in the DRH image reconstruction. It can be clearly seen that CWT could decrease the speckle or noise in DRH image and the content of information could still be displayed clearly. Moreover, the displayed image of DRH did not degrade optimally, as shown in Figure 11.

The DRH image reconstruction by CWT for $z_3 = 30$ cm was analyzed to investigate the localization of RGB spectral component, and the results are shown in Figure 12.

**Figure 11.** De-noising of DRH Image at $z_3 = 30$ cm with Level 2 as well as with the Residual: (a) Haar Wavelet; (b) Daubechies Wavelet; (c) Meyer Wavelet; (d) Coiflet Wavelet
Based on Figure 12, through the modulation of RGB spectral components using CWT based on Haar, Daubechies, Meyer, and Coiflet wavelets with level 2, the spatial frequency from the object could be successfully localized. Compared to the other CWT schemes, Meyer wavelet showed good localization of the spatial frequency.
frequency component from the object, which means in
the simulation, the scheme realized the optimum modu-
lation of the information of RGB spectral components.
The result of localization by Meyer wavelet produced a
DRH image with minimum noise and blur.

Based on the simulation results shown in Figures 5–12,
through CWT based on Haar, Daubechies, Meyer, and
Coiflet wavelets with level 2, the DRH image could be
successfully reconstructed with advantages such as de-
noising in order to minimize the speckle noise and
compress the image, while considering minimum blur
and modulation of R, G, and B spectral component. The
use of CWT to reconstruct or develop digital holograms
has many advantages compared to using Fourier
transform in the case of FFT [16–18]. The main
weakness of FFT lies on the basis of signal operation to
modulate or composing in scaling and shifting that is
referred as windowed Fourier transform (WFT). By the
virtue of a WT that has many bases of signal operation,
the modulation or composition of the signal can be
precisely developed in order to seek the highest efficiency.
Thus, WFT is not efficient enough to localize the signal
precisely as WT does. For this reason, modulation of R, G,
and B spectral components using CWT is more adaptive
and flexible. Moreover, with regard to characteristics in
signal operation, WT can achieve better compression and
de-noising than FFT as shown by the result in
Figures 10 and 11.

4. Conclusions

From the simulations, using CWT to modulate R, G,
and B spectra from an object in the DRH recording
process enhanced the reconstructed DRH image. As
shown in the simulation results, compression and de-
noising were realized by using CWT to simulate the
recording and reconstruction of a DRH image. Moreover,
CWT has a potential to enhance the localization of the
spatial frequency components of digital hologram as 3D
display media. Thus, the development of DRH for
modern and smart 3D display media can be improved
significantly. In the future, we will investigate the
performance of discrete WT for modulating R, G, and B
spectra in the case of DRH and also investigate the
compression and de-noising performances.

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