Abstract

The paper deals with the modelling of seismo-electromagnetics disturbance on the crustal earth by use of three-dimensional (3D) finite-difference time-domain (FDTD) method. The model is built up by discretizing the frontier geographical region between Java Island and Sumatra Island in a cylindrical coordinate system-based 3D object. The proposed method is applied to compute and analyze electromagnetics (EM) fields of the observed very low frequency (VLF) wave used for the investigation. Boundary condition of uniaxial perfectly matched layer (UPML) are applied surrounding the area of computation for truncating the object of simulation. The investigation are focused on the propagation time of observed VLF wave and its amplitude variation between the observation point and disturbance pulse. The result shows that the propagation time is significantly affected by the distance of observation point and the permittivity of propagation medium. Meanwhile, the addition pulse associated with the earthquake influences the amplitude of observed VLF wave instead of its frequency.

1. Introduction

During 2018 there were 382 Earth Quakes (EQs) with magnitude greater than or equal to 5 scale of Richter (SR) happened in Indonesia [1]. The two most recent dreadful EQs happened in Lombok, August 2018 causing around 500 people died and in Palu, October 2018 which was also followed by tsunami and killed around 2000 people [2]–[3]. Those situations make Indonesia as a country with high EQs potentials. Therefore, EQs prediction is required as an early mitigation. Formerly, EQs prediction could only be determined by seismic activities. However, in the past decades, research shows that the EM phenomena could also be used as EQs precursors [4]–[7].
The EM phenomena associated with EQs is commonly known as seismo-electromagnetics. An observation in [8] shows that when strong EQs happened, both satellite and ground observation detected wide-frequency band of EM waves. Some satellites e.g. GEOS-2, DE2, Intercosmos-19 and 24, indicated several cases of EM perturbations correlated with EQs have also been reported [9]. Moreover, in a mission to observe seismo-electromagnetics phenomena, China has launched China Seismo-Electromagnetic Satellite (CSES) on February 2018 [10]. Seismo-electromagnetic phenomena is usually observed in Very Low Frequency (VLF) wave. The transient and anomaly of VLF wave in ionosphere were found as an indication to EQs. A number of researches have been done to observe the correlation of EM waves propagation. The method uses time and space discretization which is derived from a differential form of Maxwell’s equations as given in Eqs. (1)–(2) [19]. \( E \) and \( H \) define electric field and magnetic field, respectively, where \( \mu_0 \) and \( \varepsilon_0 \) are permeability and permittivity of medium. The basic unit in FDTD method is called as Yee cell- an honor of a scientist Yee who firstly proposed the method in 1960 [20]. The positioning of electric and magnetic fields in each unit cell of 3D cylindrical coordinate system follows the indexing system as illustrated in Fig. 1.

Updating equations for magnetic field in FDTD notation based on cylindrical coordinate system are formulated in Eqs. (3)–(5), while the equations for updating electric field are given in Eqs. (6)–(8) [13]. Equations for updating magnetic field are discretized in time at \( n \), therefore the equations use the time difference of \( H \) field at time \( n + \frac{1}{2} \) and \( n - \frac{1}{2} \) whereas the equations for updating electric component are discretized at \( n + \frac{1}{2} \) and \( n \) so that the time difference used for the equations is at times \( n + 1 \) and \( n \).

\[
\frac{E_z^n_{i,j,k+1/2} - E_z^n_{i,j,k}}{\Delta z} = \frac{\mu}{\Delta t} \left[ H_y^{n+1/2}_{i+1/2,j,k} - H_y^{n-1/2}_{i+1/2,j,k} \right]
\]

\[
\frac{\Delta H_z}{\Delta t} = \frac{\mu}{\Delta t} \left[ H_y^{n+1}_{i+1/2,j,k} - H_y^{n-1}_{i+1/2,j,k} \right]
\]

\[
\frac{\Delta H_z}{\Delta t} = \frac{\mu}{\Delta t} \left[ E_x^{n+1}_{i,j+1,k} - E_x^{n}_{i,j+1,k} \right]
\]

Recently, the use FDTD method especially in three dimensional (3D) based on cylindrical coordinate system has been implemented to study and model the crustal earth [14]–[16]. An observation of seismo-electromagnetics phenomena using 2D FDTD method has also been reported [17]. To advance the previous research in [17],[18], this paper deals with the modelling of seismo-electromagnetics disturbance on crustal earth using 3D FDTD based on cylindrical coordinate system. The propagation time between the observation point and disturbance is investigated intensively. Meanwhile the amplitude variation of observed VLF wave as an effect of additional pulse associated with EQs is also analyzed.

2. Methods and Modelling

\[ \nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \]  

\[ \nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} \]
\[ \frac{E_p^{n}|_{i,j,k} - E_p^{n}|_{i,j,k}}{\rho \Delta \varphi} = \rho \frac{E_p^{n+1}|_{i,j,k} - E_p^{n}|_{i,j,k}}{\rho \Delta \varphi} \left( H_x^{n+1}|_{i,j,k+1} - H_x^{n-1}|_{i,j,k+1} \right) \]  
\[ \frac{\rho H_p^{n+1}|_{i,j,k} - H_p^{n+1}|_{i,j,k}}{\Delta \varphi} - \frac{\rho H_p^{n+1}|_{i,j,k+1} + 1 - H_p^{n+1}|_{i,j,k+1}}{\Delta \varphi} = \frac{\Delta H_p^{n+1}|_{i,j,k}}{\Delta \varphi} 
\]  
\[ \left( H_y^{n+1}|_{i+1,j,k} - H_y^{n-1}|_{i+1,j,k} \right) + \left( \frac{2 \varepsilon_0 \sigma \Delta t}{2 \varepsilon_0 k_0 + \sigma \Delta t} \right) \left( \frac{H_z^{n+1}|_{i,j,k+1} - H_z^{n+1}|_{i,j,k+1}}{\Delta \varphi} \right) 
\]  
\[ B_r^{n+1}|_{i,j+1/2,k+1/2} \]  
\[ \left( 2 \varepsilon_0 k_0 - \sigma \Delta t \right) \left( \frac{E_r^{n+1}|_{i,j+1/2,k+1/2}}{2 \varepsilon_0 k_0 + \sigma \Delta t} \right) 
\]  
\[ \left( H_i^{n+1}|_{i,j+1/2,k+1/2} \right) 
\]  
\[ \nabla \times H = j w \varepsilon S E \]  
\[ \nabla \times E = -j w \mu S H \]  

Implementation of electric and magnetic fields in UPML is based on the correlation between electric flux \( D \) and magnetic flux \( B \) in which these are also related to \( E \) and \( H \), respectively. The formulas for calculating \( D_r, B_r, E_r, \) and \( H_y \) in FDTD notation are expressed in Eqs. (11)-(14). \( \mu_0 \) and \( \varepsilon_0 \) are permeability and permittivity of medium, respectively, where \( k \) is constant and \( \sigma \) is lossy medium conductivity which its value depends on the axis.
geographical region between Sumatra and Java Island was chosen because it could represent various medium such as dry land, sea water and wet land. The model was divided into small element as a unit cell with the size of $\Delta \rho = \Delta z = 1 \text{ km}$ and $\Delta \phi = 0.15^\circ$. Hence, the total number of cells are $40 \times 200 \times 220$

UPML is used to model the boundary condition surrounding the area of computation with the amount cell of 10, so that the total size of simulation object is 20 km x 180 km x 200 km. The model configuration is then programmed using C++ with the visualization using excel and MATLAB. The frontier geographical map applied to model the seismo-electromagnetic phenomena is shown in Fig. 2.

For modelling the real condition, the object of simulation is divided into three parts, dry-land for modelling Sumatra Island, wet-land for modelling Java Island and sea-water for modelling Sunda straits with the characteristic medium of each summarized in Table 1. Meanwhile, the object of simulation is shown in Fig. 3 and the simulation flowchart for computation is depicted in Fig. 4.

![Figure 2. Frontier Geographical Map between Java Island and Sumatra Island for Seismo-electromagnetic Modelling](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Conductivity (S/m)</th>
<th>Permittivity coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry-land</td>
<td>0.001</td>
<td>15</td>
</tr>
<tr>
<td>Wet-land</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Sea-water</td>
<td>0.01</td>
<td>70</td>
</tr>
</tbody>
</table>

![Figure 3. Object Simulation](image)

![Figure 4. Simulation Flowchart for Computation](image)

3D FDTD method is implemented to model and simulate the seismo-electromagnetic on crustal earth. The VLF wave in atmosphere layer will be observed when the Gaussian pulse associated with earthquake is generated. The analysis of seismo-electromagnetic
FDTD Modeling for Seismo-Electromagnetics Disturbance

The model will be conducted in two scenarios; the first is to observe the effect of disturbance pulse to the VLF wave, meanwhile the second is to analyze the impact of medium propagation to the propagation of disturbance pulse which is associated with EQs. Three observation points are employed for observing both scenarios. The observation points are located at $90 \, \text{km}$ in $\varphi - \text{axis}$ and $1 \, \text{km}$ above the surface with three different distances in $z$-axis, $50 \, \text{km}$, $110 \, \text{km}$ and $170 \, \text{km}$, respectively.

There are three conditions for observing the effect of amplitude coming from the disturbance pulse to the VLF wave. The first condition is to model the excitation pulse nearby the observing point with the amplitude of $100 \, \text{V/m}$. The excitation point is positioned at the depth of $1 \, \text{km}$ below the surface, $91 \, \text{km}$ in $\varphi - \text{axis}$ and $40 \, \text{km}$ in $z$-axis. To investigate the effect of pulse excitation magnitude to the observed VLF amplitude, the second condition is to model the same excitation point with the amplitude of $1000 \, \text{V/m}$. Moreover, the last condition is to model the disturbance pulse with the amplitude of $100 \, \text{V/m}$ in a further distance located at $2 \, \text{km}$ below the surface, $92 \, \text{km}$ in $\varphi - \text{axis}$ and $40 \, \text{km}$ in $z$-axis. The purpose of the third condition is to analyze the effect of pulse excitation location to the observed VLF amplitude.

To analyze the impact of pulse’s excitation medium to the observed propagation wave, the model is situated in two conditions. The first condition is to generate the disturbance pulse at the depth of $1 \, \text{km}$ below the seawater surface with the distance of $91 \, \text{km}$ in $\varphi - \text{axis}$ and $100 \, \text{km}$ in $z$-axis. The amplitude of disturbance pulse is $100 \, \text{kV/m}$. The second condition is to model the disturbance pulse generated from wet-land at the depth of $1 \, \text{km}$ and in the distance of $91 \, \text{km}$ in $\varphi - \text{axis}$ and $160 \, \text{km}$ in $z$-axis with the amplitude of $10 \, \text{kV/m}$.

3. Results and Discussion

Effect of Adding Disturbance Pulse associated with EQs to The Amplitude of Propagation Wave. The model will be analyzed before and after the excitation of Gaussian pulse associated with EQs. The VLF sinusoidal wave is generated at $1 \, \text{km}$ above the surface with the distance of $90 \, \text{km}$ in $\varphi - \text{axis}$ and $20 \, \text{km}$ in $z$-axis. The excited VLF wave source is $E_{\varphi}$ propagates in $z$-direction with the amplitude of $1 \, \text{mV/m}$ and the frequency of $10 \, \text{kHz}$. The generated disturbance pulse is also $E_{\varphi}$ and propagates in $z$-direction at $t = 0.4651 \, \text{ms}$ with the peak happen at $0.4984 \, \text{ms}$. The VLF wave source and Gaussian pulse are shown in Figs. 5 and 6, respectively. It clearly demonstrates that the additional Gaussian pulse increases the amplitude of observed VLF wave as plotted in Fig. 7. From the result, it is acquired that the change in amplitude of the disturbance pulse is around $7.34 \times 10^{-7} \, \text{V/m}$ with the error value to the theoretical result of around $2.56 \%$.

![Figure 5. VLF Wave Source](image)

![Figure 6. Gaussian Pulse Associated with EQs](image)

![Figure 7. Amplitude Comparison of Propagation Wave with and without Disturbance Pulse in Dry-land](image)
As already stated before, the observation point took place in three different points located at $50\ km$ in $z$-axis, $110\ km$ in $z$-axis, and $170\ km$ in $z$-axis, respectively. The shape of propagation wave at these points can be seen in Fig. 8. It shows that the amplitude at $z = 50\ km$ has more considerable change followed by the amplitude at $z = 110\ km$ and $z = 170\ km$, respectively. This occurs due to the position consequence between the distance of excitation source and observation point. It can be inferred that the further the distance between the observation point and excitation source, the smaller the change in the amplitude.

Furthermore, the results in Fig. 9 shows that the amplitude variation of disturbance pulse associated with EQs has a significant impact to the propagation wave. It is acquired from the result that when the disturbance pulse amplitude of $100\ V/m$, the maximum amplitude of propagation wave is $4.81\ mV/m$. In addition, when the amplitude of disturbance pulse is increase by 10 times, the maximum amplitude of propagation wave at the observation point is $48.6\ mV/m$. Therefore, it can be concluded that the amplitude of propagation wave is proportionally increase to the addition of disturbance pulse amplitude.

Table 2. Maximum Observed Propagation Wave Amplitude from Three Different Conditions

<table>
<thead>
<tr>
<th>Excitation location</th>
<th>Pulse amplitude (V/m)</th>
<th>Observed propagation wave amplitude (mV/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby excitation point</td>
<td>100</td>
<td>4.81</td>
</tr>
<tr>
<td>Further excitation point</td>
<td>1000</td>
<td>48.6</td>
</tr>
<tr>
<td>Further excitation point</td>
<td>100</td>
<td>0.21</td>
</tr>
</tbody>
</table>

The location of disturbance pulse excitation has also a considerably impact to the observed propagation wave as depicted in Fig. 10. In compare to the first excitation point of disturbance pulse, the second point located at $92\ km$ in $\phi$ – axis, $40\ km$ in $z$ –axis and in the depth of $2\ km$ below the surfaces provides a weaker amplitude. This happens due to the longer distance between the observation point and the disturbance pulse which leads to the bigger attenuation in propagation wave. The maximum amplitude of observed propagation wave in previous three conditions are given in Table 2.

As indicated in Figs. 11-12, the medium where the disturbance pulse is generated also has a remarkably effect to the amplitude of propagation wave. Fig. 11 shows the amplitude comparison of propagation wave affected by the disturbance pulse generated in the seawater with the amplitude of $100\ kV/m$. In comparison to the first model of disturbance pulse generated in dryland, the amplitude of disturbance pulse is greater, however the maximum amplitude of propagation wave is smaller. This consequence is evoked by the higher attenuation from sea-water. From the simulation, it is observed that the change in amplitude of disturbance pulse is $5.82 \times 10^{-14}\ V/m$ with the error value to the theoretical calculation of around $3.32\ %$. 

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Meanwhile as plotted in Fig. 12, the disturbance pulse generated in wet-land with the amplitude of $10 \times 10^{-8}$ V/m has also an impact to the propagation wave. In this model, the disturbance pulse is generated 100 times higher than the amplitude of disturbance pulse in dry-land. Yet, the change in amplitude of observed propagation wave only 10 times higher than the result as depicted in Fig. 7. This happens due to the conductivity characteristic of wet-land which is higher than the conductivity of dry-land. The change in amplitude of disturbance pulse is $3.04 \times 10^{-8}$ V/m with the error value to the theoretical approach of around 0.18 %.

Summary for the change in amplitude of disturbance pulse in three mediums is given in Table 3.

### Table 3. Change in Amplitude of Disturbance Pulse in Three Mediums

<table>
<thead>
<tr>
<th>Medium excitation</th>
<th>The change in amplitude of disturbance pulse (V)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry-land</td>
<td>$7.34 \times 10^{-7}$</td>
<td>2.56</td>
</tr>
<tr>
<td>Wet-land</td>
<td>$3.04 \times 10^{-8}$</td>
<td>0.18</td>
</tr>
<tr>
<td>Sea-water</td>
<td>$5.82 \times 10^{-14}$</td>
<td>3.32</td>
</tr>
</tbody>
</table>

### Effect of Adding Disturbance Pulse associated with EQs to The Frequency of Propagation Wave

As shown in Fig. 13, the frequency of observed VLF wave is unaffected by the excitation of disturbance pulse which is associated with EQs. This happens due to the absence of frequency component in the disturbance pulse that can influence the frequency of propagation wave.

### Table 4. Propagation Time of Disturbance Pulse

<table>
<thead>
<tr>
<th>Medium observation</th>
<th>Theoretical time (ms)</th>
<th>Simulation time (ms)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry-land</td>
<td>0.5727</td>
<td>0.5726</td>
<td>0.0025</td>
</tr>
<tr>
<td>Wet-land</td>
<td>0.6000</td>
<td>0.6005</td>
<td>0.008</td>
</tr>
<tr>
<td>Sea-water</td>
<td>0.6494</td>
<td>0.6490</td>
<td>0.0062</td>
</tr>
</tbody>
</table>

**Propagation Time of Disturbance Pulse Associated with EQs.** Based on the simulation results as given in Table 4, it shows that the propagation time required by the disturbance pulse from the excitation point in dry-land to arrive at the observation point located at $90 \ km$ in $\varphi$ – axis, 1 km above the surfaces and 50 km in $z$-axis is 0.5726 ms, while the theoretical result is 0.5727 ms, so that the error value is around 0.0025 %. Furthermore, the theoretical calculation and simulation result of propagation time observed at 50 km in z-axis is 0.6494 ms and 0.6490 ms, respectively. The error value obtained in this observation point is around 0.0062%. The last observation point which is located at $z = 170 \ km$ has simulated propagation time of 0.6005 ms, while the theoretical approach has 0.6000 ms. Hence, the error value from the last observation point is around 0.008%. These results are satisfying theoretical prediction where the propagation wave...
medium with larger permittivity leads to yield smaller propagation velocity so that the propagation time becomes longer.

4. Conclusion

3D FDTD method for modelling the seismo-electromagnetics disturbance on crustal earth has been presented. The results have demonstrated that the additional disturbance pulse associated with EQs has a consequence only to the amplitude of VLF wave instead of its frequency. The amplitude of disturbance pulse had proportionally affected to the amplitude of VLF wave. It has been also demonstrated that the position of observation point has a significant influence on the amplitude variation of propagation wave. Furthermore, the conductivity of propagation medium has contributed to the attenuation of propagation wave which lead to lower the observed wave amplitude. Meanwhile the permittivity of propagation medium has affected the propagation time of disturbance pulse to get in the observation point. In addition, the propagation time has a dependency not only on the permittivity of propagation medium but also on the distance of observation point.

References